# Julia Robinson and Hilbert's Tenth Problem

*Reviewed by Carol Wood*


**Julia Robinson and Hilbert's Tenth Problem**
*Zala Films*
*Produced and directed by George Csicsery*

This film by George Csicsery is an hour-long documentary on the life and work of Julia Robinson, an extraordinary mathematician who played a key role in the solution of Hilbert's Tenth Problem. Csiscery is perhaps best known in the mathematical community for his documentary about Paul Erdős, *N is a Number*. The final version of Csicsery's latest work premiered at the Joint Mathematics Meetings in San Diego on January 7, 2008, to an appreciative audience. The setting was particularly appropriate: Julia moved with her family to San Diego when she was a small girl, and she remained there through her early years of college. The film includes a 1920s view of a pristine bay, now the site of the convention center and meeting hotels.

The documentary was many years in the making and draws on various archival materials, both film and still shots, plus interviews of colleagues and friends of the Robinsons, including the other key figures involved in the solution of Hilbert's Tenth Problem (H10): Martin Davis, Hilary Putnam, and Yuri Matijasevich. H10 asks whether there is an algorithm for determining when a polynomial equation with integer coefficients has an integer solution (for details, see Bjorn Poonen's article in the April 2008 *Notices*). The film is narrated sweetly by Danica McKellar, actress (Winnie in *The Wonder Years*), math major (at the University of California, Los Angeles), and author of a book about mathematics aimed at girls (*Math Doesn't Suck*).

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Csicsery intertwines for us a human story and a mathematical story. In Julia Robinson we find a mathematician who was a heroine in her own time and a role model for all time. It is a story of childhood, illness, love, marriage, disappointment, obsession, and triumph. It is filled with extraordinary instances of luck both good and bad. The stars of the film are the incandescent Bowman sisters, Julia Robinson and Constance Reid. Julia was very fortunate to have Constance as a sister and a spokesperson. Julia's sister is no stranger to the mathematical community, having written prize-winning books about mathematics and mathematicians, including a masterpiece about Hilbert. Reid's storytelling and her presence enliven the film. Her 1996 book *Julia, A Life in Mathematics*, published in the MAA Spectrum series, is a perfect companion to the film. (See also Reid's article, "Being Julia Robinson's Sister" in the December 1996 *Notices*.)

The narrative begins with Julia's childhood, and follows her mathematical and personal odyssey.

**The Bowman sisters, Julia Robinson and Constance Reid.**

### My Julia Story

In 1968 a 22-year-old graduate student and her 25-year-old mathematician husband walked through the lobby of the headquarters hotel at the San Francisco AMS meeting. This was the first mathematics meeting for either of them. Walking in the other direction was a distinguished-looking couple of an older generation. They stopped to speak to the young couple, saying, "We live in this area and decided we should come to the meeting and meet some new people. We are Julia and Raphael Robinson." The student doesn't remember any further detail of the pleasantries that were exchanged in the lobby that day, although perhaps she had the presence of mind to mention that she was taking a course in model theory from Abraham Robinson. Once out of earshot she asked her husband excitedly "Do you *know* who they are??!!"

I was that student, and I had just met two of the stars in my mathematical world. Nothing that thrilling has happened to me at AMS meetings in the intervening 40 years, nor do I expect it ever will.

—*C. W.*

**Left to right: Martin Davis, Julia Robinson, Yuri Matiyasevich.**

Opportunities for young women in mathematics were severely constrained in her day. Nonetheless, in the late 1930s she found her mathematical milieu in Berkeley. Two figures stand out in her mathematical development. Raphael Robinson was one of her first teachers at Berkeley. He then became her mentor and soon thereafter her husband. Her thesis advisor, Alfred Tarski, arrived at Berkeley during her graduate study. Both Robinsons benefited from Tarski's presence. One of Tarski's great strengths was his ability to pose powerful, insightful questions. As the story goes, many of these were communicated at the men's faculty club, some to Raphael. Fortunately, Raphael passed them along to Julia.

The main result of Julia's 1948 thesis, easily in my top ten list of favorite theorems, answered one of Tarski's questions. She proved the definability of the integers in the rationals, a result that transported Gödel's undecidability phenomenon from the integers to the rational numbers. Shortly thereafter she began to work on H10 and produced an inspired sufficient condition for a negative solution, showing that if one could find a single diophantine equation whose roots displayed exponential growth, then the full problem would be undecidable. Fulfilling the condition, called JR by others, would provide the missing piece of the solution as envisioned by Davis, Putnam, and Robinson. This was finally achieved in 1970 by Yuri Matiyasevich, to the delight of the other three. An endearing aspect of the story about H10 is the generosity of the four main players in deflecting credit toward each other. Once the solution of H10 was complete, recognition of Julia's role came swiftly and dramatically, a truly joyful part of the story Csicsery tells.

The film also takes us to an era when the speed of mathematical communication could be glacial. It is already difficult to imagine life without email, much less to recall the severe restrictions on travel and correspondence due to the cold war. Communication with many Russian colleagues was extremely difficult throughout Julia's lifetime. Yuri Matiyasevich was not allowed to travel to the U.S. They did meet in other countries, but their collaboration was done mostly via handwritten letters. Yuri's description of the challenge of mailing a letter with mathematical content to the U.S. in the 1970s is quite amazing.

Julia's life ended too early, but she will be remembered as she wished to be, for the mathematics that she did. Her work has a modern quality, with its emphasis on definability and on the connections between number theory and logic. Her collected works, edited by Solomon Feferman, were published by the AMS in 1996. The film is made all the more timely by current activity related to H10, e.g., by Poonen's recent improvement of Julia's thesis. Poonen's result reduces the complexity of the formula needed to select the integers from among the rationals, bringing us closer to the desired analogue of H10 for the rational numbers; how close remains to be seen.

**Julia and Raphael Robinson.**

I cannot resist a word here about Raphael Robinson, something I believe Julia would want said. "Hillary Clinton is a politician in her own right!" sounds quaint in 2008. But surely Julia, in her role as the wife of a Berkeley professor, was regularly referred to as "a mathematician in her own right", up to and including the time of her election to the National

Academy. My point is that one should not overcompensate, while celebrating the achievements of Julia, by overlooking the related work of her husband. As luck would have it, his fame does not match hers, nor is he the subject of this film, although his crucial role in Julia's mathematical development is acknowledged. However, this kind and quiet man was very much a "mathematician in his own right", with many beautiful results to his credit, including undecidability results for certain function fields, together with an impressive record as a problem solver. He was a key figure in early work on tiling of the plane and the author of undecidability results for certain fields. One account of Raphael's work is found in the article by Leon Henkin in the *Bulletin of Symbolic Logic*, Volume 1, No. 3 (1995).

The AMS film premier was hosted by the Clay Mathematics Institute, which was a major sponsor of the film's production, as were Margaret and Will Hearst. George Csicsery, Constance Reid, and Martin Davis were present at the showing, adding to the celebratory atmosphere. Even without principal figures in attendance, this film is worth a viewing by readers of the *Notices*. It is equally suitable for an undergraduate math club and for a mathematics colloquium audience. I have not yet had the opportunity to show it to nonmathematicians, but I can imagine that a general audience, such as viewers of the *NOVA* series on PBS, would also be captivated by the story. However, this is a film about mathematics and mathematicians, and it will be most deeply appreciated by mathematicians and our students.

**Top of page: Julia and Constance at the beach. Below: Constance Reid and Julia Robinson.**

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