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AN INEQUALITY FOR MIXED L^p–NORMS

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Abstract. Consider a nonnegative measurable function f defined on $\Omega_1 \times \Omega_2$, where Ω_j is a probability space with probability measure μ_j. We prove the inequality

$$\left[\iint_{\Omega_1\times\Omega_2} f\, d\mu_1 d\mu_2\right]^p + \iint_{\Omega_1\times\Omega_2} f^p\, d\mu_1 d\mu_2 \geqslant \int_{\Omega_1}\left[\int_{\Omega_2} f\, d\mu_2\right]^p d\mu_1 + \int_{\Omega_2}\left[\int_{\Omega_1} f\, d\mu_1\right]^p d\mu_2$$

provided that $1 \leqslant p \leqslant 2$. The inequality fails in general if $p > 2$. It also fails if one of the measures μ_j has total mass greater than one. Curiously however, the inequality is true for all $p \in [1,\infty)$ if the measures μ_j are counting measures. This last fact follows from a subadditivity result proved by G. A. Raggio for p-entropies. Our inequality also has a formulation in terms of p-entropies.

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