B561 Assignment 2
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Relational Algebra

The following conventions are used in the problems:

- We write $r(ABC)$ for a relation instance with relation name r and attributes (or fields) A, B, and C. In other words, the relation r "is a relation on scheme" $R = \{A, B, C\}$. We will just write "relation" for relation instance if not otherwise noted.
- For any attribute A, $dom(A)$ denotes the domain of A.
- If not noted otherwise, it is assumed that $dom(X) \neq dom(Y)$ for all $X, Y \in \{A, B, ..., Y, Z\}$.
- $\bowtie$ (in LaTex: \bowtie) is the symbol for the natural join.

1. Let $r(ABC)$ and $s(BCD)$ be two relations, with $a \in dom(A)$ and $b \in dom(B)$. Which of the following expressions are properly formed? If an expression is not properly formed, explain briefly why.

   (a) $r \cup s$

   (b) $\pi_B(r) - \pi_B(s)$

   (c) $\sigma_{B=b}(r)$

   (d) $\sigma_{A=a,B=b}(s)$

   (e) $r \bowtie s$

   (f) $\pi_A(r) \bowtie \pi_D(s)$

2. Let the relation instances for r and s be given as follows:

A	B	C
a	b	c
a	b'	c'
a	b"	c'
a'	b'	c

B	C	D
b'	c'	d
b"	c'	c'
b"	c	d

Compute the values of the following expressions.

(a) $\sigma_{A=a}(r)$

(b) all the properly formed expressions from the previous exercise.

3. Let r and s be two instances of the same relation schema with key K. Which of the following relation instances must necessarily have key K? Explain briefly why.

(a) $r \cup s$

(a) $r \cap s$

(a) $r - s$

(a) $\pi_K(r)$

(a) $r \bowtie s$

4. Let $r(R)$ be a relation (note: here R is a set of attributes) with $A \in R$ and let $a, a' \in dom(A)$. Prove

$$[\sigma_{A=a,A=a'}(r) = \emptyset \text{ or } \sigma_{A=a,A=a'}(r) = \sigma_{A=a}(r)].$$

5. Let $r(\underline{A}BC)$ be a relation (i.e., A is a key). What can be said about the size of $\sigma_{A=a}(r)$.

6. Let X be a subset of R (both are sets of attributes), $A \in R, A \notin X$, and let r be a relation on R (i.e., r is the relation $r(R)$). Find a counterexample to

$$\pi_X(\sigma_{A=a}(r)) = \sigma_{A=a}(\pi_X(r)).$$

7. Let X be a subset of R and let r and s be relations on R (i.e., r is the relation $r(R)$ and s is the relation $s(R)$). Prove or disprove the following equalities.

(a) $\pi_X(r \cap s) = \pi_X(r) \cap \pi_X(s)$

(b) $\pi_X(r \cup s) = \pi_X(r) \cup \pi_X(s)$

(c) $\pi_X(r - s) = \pi_X(r) - \pi_X(s)$

8. Let r and r' be relations on R, and let s be a relation on S. Prove or disprove:

   (a) $(r \cap r') \bowtie s = (r \bowtie s) \cap (r' \bowtie s)$

   (b) $(r - r') \bowtie s = (r \bowtie s) - (r' \bowtie s)$

9. Given relations $r(R), s(S)$ and $q = r \bowtie s$, define $r' = \pi_R(q)$ and $s' = \pi_S(q)$. Prove

$$q = r' \bowtie s'.$$

10. Given a relation $q(RS)$, find a sufficient condition for

$$q = \pi_R(q) \bowtie \pi_S(q).$$

    Is your condition necessary?

11. Let $r(R)$ and $s(S)$ be relations where $R \cap S = \emptyset$. Prove

$$(r \bowtie s) \div s = r.$$

    Here $\div$ denotes the division operator.

12. Let r be a relation on scheme R and let s and s' be relations on scheme S, where $R \supseteq S$. Show that if $s \subseteq s'$, then

$$r \div s \supseteq r \div s'.$$

    Show the converse is false.

13. Let $r(R)$ and $s(S)$ be relations with $R \supseteq S$ and let $R' = R - S$. Note that $t \in s$ denotes a tuple t in s and the expression $\sigma_{S=t}(s)$ denotes the selection of exactly the tuple t on s. Prove the identities

   (a) $r \div s = \pi_{R'}(r) - \pi_{R'}((\pi_{R'}(r) \bowtie s) - r)$

   (b) $r \div s = \bigcap_{t \in s} \pi_{R'}(\sigma_{S=t}(r))$

14. Show that any equijoin can be specified in terms of natural join and renaming, given sufficient extra attributes with the correct domains.