# A Methodology for Coupling Fragments of XPath with Structural Indexes for XML Documents 

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## Indices for XML Dała

* XPath: expressions specify patterns
* Expression evaluation is aided by indices:
- value-based: consider node values
- structure-based: values \& document structure


## What about techniques for using structural indices in query evaluation?

(1) For which fragments of XPath are particular structural indices ideally suited?
e.g., in the relational world, range queries and $B$-trees
(2) For these fragments, how are its expressions optimally evaluated with the index?
(3) Can the answers to (1) \& (2) be bootstrapped to provide general techniques for evaluation of arbitrary XPath expressions with indices?

## In this paper ...

* Develop general framework and methodology for investigating pairings of query languages and structural indexes
* Illustrate this methodology on important special case of XPath and A(k)/P(k) indexes


## Let's focus on (1):

For which class of XPath expressions are the P(k) partitions ideally suited?

## A(k) Indices: Localized Bisimilarity

* 1-Index/Dataguide much too fined-grained, and hence too large for practical use ...
* Kaushik et al (ICDE '02) proposed restricting 1-index to "k-neighborhood"
* Substantially smaller than 1-index/ DataGuide
* Distinguishes nodes by labels and incoming paths of length $k$


## Data Model $D=(V, E d, r, \lambda)$

* Documents are finite unordered node-labeled trees:
- nodes $V$
- edges $E d \subseteq V \times V$
- root $r \in V$
- labels $\lambda: V \rightarrow \mathcal{L}$


## The A(k) Partition of a Document

$$
n_{1} \equiv A(k) n_{2}
$$

* For nodes $n l$ and $n 2$, we have that they are $A(k)-$ equivalent if
- they have the same label, and
- for $k>0$, if one has a parent, so does the other and, furthermore, their parents are A(k-1)equivalent
* The partition induced by this relation on nodes is called the A(k) partition of the document




## Consider A(k) indices on the "Design" department subtree



## A(O) index on "Design" department subtree



## A(1) index on "Design" department subtree



## A(2) index on "Design" department subtree

## The P(k) Partition of a Document

$$
\left(n_{1}, m_{1}\right) \equiv_{P(k)}\left(n_{2}, m_{2}\right)
$$

* For nodes $\mathrm{nl}, \mathrm{ml}, \mathrm{n} 2$, and m 2 we have that $(\mathrm{nl}, \mathrm{ml})$ and $(\mathrm{n} 2, \mathrm{~m} 2)$ are $\mathrm{P}(\mathrm{k})$-equivalent if
- (nl, ml) and (n2, m2) are in UpPaths(0,k)
- the distance from nl to ml in the document is the same as that from $n 2$ to $m 2$, and
- $n_{1} \equiv_{A(k)} n_{2}$
* The partition induced by this relation on node pairs in UpPaths( $D, k)$ is called the P(k) partition of the document


## XPath Algebra

$$
D=(V, E d, r, \lambda)
$$

$$
\begin{aligned}
\varepsilon(D) & =\{(m, m) \mid m \in V\} \\
\emptyset(D) & =\emptyset \\
\downarrow(D) & =E d \\
\uparrow(D) & =E d^{-1} \\
\ell(D) & =\{(m, m) \mid m \in V \text { and } \lambda(m)=\ell\}
\end{aligned}
$$

# XPath Algebra <br> $$
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\ell(D) & =\{(m, m) \mid m \in V \text { and } \lambda(m)=\ell\} \\
E_{1} \cup E_{2}(D) & =E_{1}(D) \cup E_{2}(D) \\
E_{1} \cap E_{2}(D) & =E_{1}(D) \cap E_{2}(D) \\
E_{1}-E_{2}(D) & =E_{1}(D)-E_{2}(D) \\
E_{1} \circ E_{2}(D) & =\left\{(m, n) \mid \exists w:(m, w) \in E_{1}(D) \&(w, n) \in E_{2}(D)\right\} \\
E_{1}\left[E_{2}\right](D) & =\left\{(m, n) \in E_{1}(D) \mid \exists w:(n, w) \in E_{2}(D)\right\}
\end{aligned}
$$

## XPath Algebra

$$
D=(V, E d, r, \lambda)
$$

* Global semantics of expressions: binary relation over $V$
* Local semantics of expressions: for $m \in V$

$$
E(D)(m)=\{n \in V \mid(m, n) \in E(D)\}
$$

## XPath Algebra



## "Retrieve all department names"

## XPath Algebra



## "Retrieve all department names"

$E=$ Projects $\circ \downarrow \circ$ Department $\circ \downarrow \circ$ Name

## XPath Algebra



## "Retrieve all department names"

$E=$ Projects $\circ \downarrow \circ$ Department $\circ \downarrow \circ$ Name

$$
E(D)=\left\{\left(n_{0}, n_{3}\right),\left(n_{0}, n_{7}\right)\right\}
$$

## XPath Algebra



## "Retrieve all department names"

$E=$ Projects $\circ \downarrow \circ$ Department $\circ \downarrow \circ$ Name

$$
\begin{gathered}
E(D)=\left\{\left(n_{0}, n_{3}\right),\left(n_{0}, n_{7}\right)\right\} \\
E(D)\left(n_{0}\right)=\left\{n_{3}, n_{7}\right\}
\end{gathered}
$$

## XPath Algebra



## XPath Algebra



## "Retrieve all projects which are sub-projects of projects with a website"

$$
E=\text { Project }[\uparrow \circ \text { Project } \circ \downarrow \circ \text { Web }]
$$

## XPath Algebra



## "Retrieve all projects which are sub-projects of projects with a website"

$$
\begin{gathered}
E=\text { Project }[\uparrow \circ \text { Project } \circ \downarrow \circ \text { Web }] \\
E(D)=\left\{\left(n_{17}, n_{17}\right),\left(n_{18}, n_{18}\right)\right\}
\end{gathered}
$$

## Upward-k Algebras

Upward-k Algebras: for $k>=0, U(k)$ is the fragment of the XPath-Algebra with expressions that do not use the $\downarrow$ primitive and have at most $k$ uses of the $\uparrow$ primitive in a "path"

## Upward-k Algebras


"Retrieve sub-project leaders"

$$
E=\operatorname{Lead}[\uparrow \circ \text { Project } \circ \uparrow \circ \text { Project }]
$$

## Upward-k Algebras


"Retrieve sub-project leaders"

$$
E=\operatorname{Lead}[\uparrow \circ \text { Project } \circ \uparrow \circ \text { Project }]
$$

## In U(2) but not in U(1)!

## Language Indistinguishability

$$
\left(n_{1}, m_{1}\right) \equiv_{\mathcal{F}}\left(n_{2}, m_{2}\right)
$$

For fragment $\mathcal{F}$ of the XPath algebra, we say node pairs (nl, m1) and ( $n 2, \mathrm{~m} 2$ ) are indistinguishable by $\mathcal{F}$ if for any expression $E$ in $\mathcal{F}$, it is the case that $\left(n_{1}, m_{1}\right) \in E(D) \Longleftrightarrow\left(n_{2}, m_{2}\right) \in E(D)$

## Coupling P(k) and U(k) Indistinguishability

## Coupling Theorem:

Let $D$ be a document and $k \in \mathbb{N}$. The $P(k)$-partition of $D$ and the $U(k)$-partition of $D$ are the same.

## Proof:

* P(k) indistinguishability implies U(k) indistinguishability. via induction on U(k) expressions
* P(k) distinguishability implies U(k) distinguishability, via construction of partition-block labeling expressions


## Coupling P(k) and U(k) Indistinguishability

## Block-Union Theorem:

Let $D$ be a document, $k \in \mathbb{N}$, and $E \in U(k)$. Then there exists a class $\mathfrak{B}_{E}$ of partition blocks of the $P(k)$-partition of $D$ such that $E(D)=\bigcup_{B \in \mathfrak{B}_{E}} B$.
... follows directly from Coupling Theorem
These results provide a precise linguistic characterization of $A(k)$ and $P(k)$ partitions, in answer to question (1).

## Now let's consider question (2):

How are U(k) expressions to be evaluated with the help of P(k) partitions?

## Upward Algebra Eval

* for expressions in U(k), direct look-up in P(k) index
* for expressions in U(I), I>k, then by decomposition into U(k) sub-expressions and joining sub-results


# Finally, let's consider question (3): 

Can these results be bootstrapped to provide general techniques for evaluation of full XPath?

## XPath Algebra Eval

* Via predicate elimination and inversion of remaining "downward" subexpressions into Upward-Algebra subexpressions:

$$
\begin{aligned}
E & \rightarrow E^{-1} \\
\hline \epsilon & \rightarrow \epsilon \\
\emptyset & \rightarrow \emptyset \\
\downarrow & \rightarrow \uparrow \\
\hat{\lambda} & \rightarrow \hat{\lambda} \\
E_{1} \cup E_{2} & \rightarrow E_{1}^{-1} \cup E_{2}^{-1} \\
E_{1} \cap E_{2} & \rightarrow E_{1}^{-1} \cap E_{2}^{-1} \\
E_{1}-E_{2} & \rightarrow E_{1}^{-1}-E_{2}^{-1} \\
E_{1} \diamond E_{2} & \rightarrow E_{2}^{-1} \diamond E_{1}^{-1} .
\end{aligned}
$$

* then proceed as before with Upward-Algebra eval


## XPath Algebra Eval

Suppose we have a document $D$, the $P(2)$ partition of $D$, and the query $\downarrow[\downarrow]$

## XPath Algebra Eval

Suppose we have a document $D$, the $P(2)$ partition of $D$, and the query $\downarrow[\downarrow]$
then ...
$\downarrow[\downarrow](D)$

## XPath Algebra Eval

Suppose we have a document $D$, the $P(2)$ partition of $D$, and the query $\downarrow[\downarrow]$
then ...

$$
\begin{gathered}
\downarrow[\downarrow](D) \\
\downarrow \\
\downarrow \diamond \downarrow \uparrow(D)
\end{gathered}
$$

## XPath Algebra Eval

Suppose we have a document $D$, the $P(2)$ partition of $D$, and the query $\downarrow[\downarrow]$
then ...


## XPath Algebra Eval

Suppose we have a document $D$, the $P(2)$ partition of $D$, and the query $\downarrow[\downarrow]$
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## XPath Algebra Eval

Suppose we have a document $D$, the $P(2)$ partition of $D$, and the query $\downarrow[\downarrow]$
then ...

... and this can be evaluated directly with the P(2) parition

## Research Directions

* Currently developing new data structures leveraging these results
- develop fast P(k) partition block look-up algorithms
* Further study of query decomposition and inversion algorithms
* Study workload driven index creation
* Study localized branching-path queries and develop appropriate index structures


## Thanks!

Questions?

