B561 Review Questions for Functional Dependencies, Normal Forms, Decompositions

Dirk Van Gucht & Mathias Niepert

October 25, 2007

(1) Consider the relation schema $R(A, B, C, D, E, F, G, H)$ with functional dependencies $\{BE \rightarrow GH, G \rightarrow FA, D \rightarrow C, F \rightarrow B\}$.

   (a) Find a (minimal) key for R. Is there any key of R that does not contain the attribute D? Explain.

   (b) Is the schema currently in BCNF? Explain.

   (c) Use one step of the BCNF decomposition to decompose R into two subrelations. Are the subrelations in BCNF?

   (d) Show that your decomposition from part (c) is lossless.

   (e) Is your decomposition from part (c) dependency preserving?

   (f) Continue the decomposition until you obtain a BCNF decomposition of R. Is your final decomposition dependency preserving?

(2) Consider the relational schema R = (A, B, C, D, E, F) with set of FDs F = $\{A \rightarrow CD, AC \rightarrow DF, F \rightarrow E, B \rightarrow A\}$.

   (a) Is R in Boyce-Codd Normal Form (BCNF)? If not, produce a lossless decomposition of R into BCNF. Show your work. How can you be sure your decomposition is lossless?

   (b) Is your decomposition from part (a) dependency preserving? Briefly justify your answer.

(3) Consider a relation schema $R(A, B, C, D, E, F, G, H)$, with functional dependencies $\{FH \rightarrow HGB, GA \rightarrow CD, C \rightarrow E, B \rightarrow D, FG \rightarrow D\}$.

(a) Find a minimal cover for the FDs.

(b) Is the dependency FHA GB implied by the FDs?

(c) Is the decomposition into FHGAB and GABCDE lossless?

(d) Is the above decomposition dependency preserving?

(e) Is there a lossless, dependency preserving decomposition of this schema into BCNF? If so, exhibit one. If not, explain.

(4) Consider the relation schema $R(A, B, C, D, E, F, G)$ and accompanying set of functional dependencies $F = \{A \rightarrow D, ADG \rightarrow F, ACE \rightarrow BD, B \rightarrow C, C \rightarrow A, D \rightarrow G, E \rightarrow B, EF \rightarrow AD, F \rightarrow E, G \rightarrow F\}$.

(a) Show that A is a key (i.e., minimal superkey) for R.

(b) Give a lossless-join, dependency preserving decomposition into BCNF for R.

(c) Argue that no 2-attribute subset of $\{A, B, C, D, E, F, G\}$ is a key for R.

(5) Using only Armstrong's Axioms and the FDs $AB \rightarrow C, A \rightarrow BE, C \rightarrow D$, give a complete derivation of the FD $A \rightarrow D$.

(6) Show that the following inference rules are derivable from Armstrong's axioms (i.e., are sound rules for functional dependencies):

(1) Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$.

(2) Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$.

(3) Strong Transitivity: If $X \rightarrow Y$ and $YW \rightarrow Z$, then $XW \rightarrow Z$.

(7) Show that the following inference system is sound and complete for functional dependencies (i.e., equivalent to Armstrong's axioms):

- Reflexivity: If $X \subseteq Y$, then $Y \rightarrow X$; and
- Strong Transitivity: If $X \rightarrow Y$ and $YW \rightarrow Z$, then $XW \rightarrow Z$.