B561 Solutions for Review Questions

Dirk Van Gucht & Mathias Niepert

November 4, 2007

(1) Consider the relation schema $R(A, B, C, D, E, F, G, H)$ with functional dependencies $\mathcal{F} = \{BE \rightarrow GH, G \rightarrow FA, D \rightarrow C, F \rightarrow B\}$.

(a) Find a (minimal) key for R.

Solution

Given a relation R. A set of attributes X is key, if it is a superkey for R and for all $A \in X$, $X - \{A\}$ is not a superkey.

For a relation R, $X \subseteq R$ is a superkey, if the FD $X \rightarrow R - X$ holds or, equivalently, if the closure of X is R.

BDE is a superkey. We can use the attibute closure algorithm from the textbook (page 614) to show this. The closure of the set BDE is ABCDEFGH. Hence, BDE is a superkey. Now we can use the same algorithm to show that BD, BE, and DE are not superkeys. The closure of BD is BCD, the closure of BE is $ABEFGH$, and the closure of DE is CDE.

Is there any key of R that does not contain the attribute D? Explain.

Solution

It is sufficient to show that $R - \{D\}$ is not a superkey. The closure of $R - \{D\}$ is $ABCEFGH$, hence, neither $R - \{D\}$ nor any of its subets is a superkey. Thus, every *key* of R must contain D.

(b) Is the schema currently in BCNF? Explain.

Solution

A relation R is in BCNF is for every FD $X \rightarrow Y$ in $\mathcal{F}^+$ (the closure of the set $\mathcal{F}$ of given FDs) one of the following statements is true:

- $X \rightarrow Y$ is a trivial constraint; or

- X is a superkey.

Clearly, R is not in BCNF, since, for example, $F \to B$ is a non-trivial FD that holds in R but F is not a superkey by the previous exercise.

(c) Use one step of the BCNF decomposition to decompose R into two subrelations.

Solution We use the decomposition procedure described in the textbook on page 623. Thereto, take the BCNF violating FD $F \to B$, and decompose R into $R - \{B\} = ACDEFGH$ and FB.

Are the subrelations in BCNF?

Solution
FB clearly is in BCNF. However, $R' = ACDEFGH$ is not, since $D \to C$ is a non-trivial FD in R' but D is not a superkey, since the closure of D does not contain A, for example.

(d) Show that your decomposition from part (c) is lossless.

Solution
$ACDEFGH \cap FB = F$. The decomposition is lossless, if and only if either $F \to FB$ or $F \to ACDEFGH$, or both are elements of $\mathcal{F}^+$. Since $F \to FB$ is an element of $\mathcal{F}^+$ (augmentation with F on $F \to B$), the decomposition is lossless.

(e) Is your decomposition from part (c) dependency preserving?

Solution
(See page 621 in the textbook). The FD $BE \to GH$ is neither in the projection of $\mathcal{F}$ on $ACDEFGH$ nor in the projection of $\mathcal{F}$ on BF. In addition, it is not in the closure of the union of these two projections (again, we can use the attribute closure algorithm to show this). Hence, the decomposition is not dependency-preserving.

(f) Continue the decomposition until you obtain a BCNF decomposition of R. Is your final decomposition dependency preserving?

Solution
The next step of the decomposition process outlined in the book would decompose the relation $ACDEFGH$ into the relations $ACEFGH$ and CD. Then, since $BE \to G$ is in $\mathcal{F}^+$, $ACEFGH$ is decomposed into $ACEFH$ and BEG. $ACEFH$, BEG, BE, and FB are all in BCNF. The decomposition is cleary not de-

pendency preserving (we already showed in the previous exercise, that the first decomposition step was not dependency preserving).

(4) Consider the relation schema $R(A, B, C, D, E, F, G)$ and accompanying set of functional dependencies $F = \{A \rightarrow D, ADG \rightarrow F, ACE \rightarrow BD, B \rightarrow C, C \rightarrow A, D \rightarrow G, E \rightarrow B, EF \rightarrow AD, F \rightarrow E, G \rightarrow F\}$.

(a) Show that A is a key (i.e., minimal superkey) for R.

Solution

Using the chase method described in the textbook, we can infer that $A \rightarrow BCDEFG$. Since $\emptyset$ is never a key, A is clearly minimal.

(b) Give a lossless-join, dependency preserving decomposition into BCNF for R.

Solution

The existing relation is already in BCNF. As such, it is clearly lossless and dependency preserving.

(c) Argue that no 2-attribute subset of $\{A, B, C, D, E, F, G\}$ is a key for R.

Solution

We have the cycle of FDs A $\rightarrow$ D;D $\rightarrow$ G;G $\rightarrow$ F; F $\rightarrow$ E;E $\rightarrow$ B;B $\rightarrow$ C;C $\rightarrow$ A. Thus, using transitivity, for any attribute X $\in$ ABCDEFG, we can show that X $\rightarrow$ ABCDEFG $\in F^+$. Hence, every singleton set is a key. Thus, no 2-attribute subset can be minimal, so it cannot be a key!

(5) Using only Armstrong's Axioms and the FDs $AB \rightarrow C, A \rightarrow BE, C \rightarrow D$, give a complete derivation of the FD $A \rightarrow D$.

Solution

1. AB $\rightarrow$ C Given in problem
2. C $\rightarrow$ D Given in problem
3. AB $\rightarrow$ D Transitivity on 1 and 2
4. A $\rightarrow$ BE Given in problem
5. BE $\rightarrow$ B Reflexivity on attributes B and E
6. A $\rightarrow$ B Transitivity on 4 and 5
7. A $\rightarrow$ AB Augmentation on 6 with A
8. A $\rightarrow$ D Transitivity on 7 and 3

(6) Show that the following inference rules are derivable from Armstrong's axioms (i.e., are sound rules for functional dependencies):

(1) Union: If $X \to Y$ and $X \to Z$, then $X \to YZ$.

(2) Decomposition: If $X \to YZ$, then $X \to Y$ and $X \to Z$.

(3) Strong Transitivity: If $X \to Y$ and $YW \to Z$, then $XW \to Z$.

Solution

(1) (1) X → Y Given
(2) X → Z Given
(3) X → XY (1) and Augmentation
(4) XY → YZ (2) and Augmentation
(5) X → YZ (3) and (4) Transitivity

(2) (1) X → YZ Given
(2) YZ → Y Reflexivity
(3) YZ → Z Reflexivity
(4) X → Y (1) and (2) Transitivity
(5) X → Z (1) and (3) Transitivity

(3) (1) X → Y Given
(2) XW → YW (1) Augmentation
(3) YW → Z Given
(4) XW → Z (2) and (3) Transitivity

(7) Show that the following inference system is sound and complete for functional dependencies (i.e., equivalent to Armstrong's axioms):

- Reflexivity: If $X \subseteq Y$, then $Y \to X$; and
- Strong Transitivity: If $X \to Y$ and $YW \to Z$, then $XW \to Z$.

Solution We only have to show that we can derive Armtrong's axioms from the given inference rules (we showed the converse in the previous exercise). Trivially, transitivity can be derived by strong transitivity for $W = \emptyset$ and reflexivity is given. Augmentation can be derived as follows

(1) $X \to Y$ Given
(2) $YW \to YW$ Reflexivity
(3) $XW \to YW$ (1) and (2) Transitivity