Errata to The Analysis of Algorithms, Second Printing. 8–27–2006

Usually just the corrected segment of text is given. Negative line numbers indicate the number of lines from the bottom.

p 2, l 3. (Only for $n > 5 \times 10^{17}$ does $n^{0.1}$ become noticably larger than $\lg n$.)

p 18, ex. 4.

$$2^{\frac{\ln\left(1+\frac{t(-\ln(1-p))}{\ln 2}\right)}{-\ln(1-p)}} = \left(\frac{\ln 2 + t(-\ln(1-p))}{\ln 2}\right)^{\ln 2/(-\ln(1-p))}.$$

p 20, ex. 2. Show that the number of times you go around the loop in the Euclidean Algorithm (Algorithm 1.5) is no more than $2\lg n$.

p 21, l −12 (m in our example)

p 27, l 2. A *triangular matrix* is one in which all the elements above

p 27, ex. 2. previous exercise can be written as

p 36, l 1, Step 5 is done the same number of or one less time than Step 4 each time Step 2 is done.

p 49, l 7. order. The first element in each file has subscript

p 49, l 9. once again the first remaining element has subscript

p 49, Step 3. Steps 3–5 and 6–8 are

p 52, eq. (105). $= 1 + \max\{C_{m,n/2-2^{t-1}} + m,\ t + C_{m-1,n/2} + m - 1\}$.

p 53, eq. (106-107) $C_{m,n} = 1 + \max\{C_{m,n/2-2^{t-1}} + m,\ t + C_{m-1,n/2} + m - 1\}$

$$\begin{aligned} C_{m,n} &= 1 + \max\{C_{m,n/2-2^{t-1}} + m,\ t + C_{m-1,n/2} + m - 1\} && 106 \\ &= 1 + \max\{C_{m,n/2-2^{t-1}},\ t + C_{m-1,n/2} - 1\} + m && 107 \end{aligned}$$

provided n is even, $m \geq 2$, and $n \geq 2m + 2$. Now comes a clever portion of the proof, a crucial observation: by eq. (103) $1 + \max\{C_{m,n/2-2^{t-1}},\ t + C_{m-1,n/2} - 1\} = C_{m,n/2}$, so we have

p 53, l −9. left side of eq.

p 55, ex. 3. $C_{m,2m+1}$

p 55, l 20. A predicate P is called $\rightarrow$-complete if, whenever $P(x)$ is *true* for all x in $\delta^+(y)$, then $P(y)$ is also *true*. (In particular, $P(y)$ is *true* when $\delta^+(y)$ is empty.

p 55, l −10. A relation is *confluent* if $x \uparrow y$ implies $x \downarrow y$. Confluence is important because, in a Noetherian confluent relation,

p 61 l −1. For example, the formula $\sum_{0\leq i\leq n} a_i$ is not in closed form, because the summation sign stands for the addition of $n+1$ elements.

p 65, l −10. The second sum

p 67, No period at the end of equation 31.

p 72 l 19. in Steps 3 and 6 of Split.

p 74, TABLE 2.1

Position	*Input*	*Intermediate*				*Output*
0	A	A	A	A	A	A
1	**G**	**B**	A	A	A	A
2	C	C	B	B	B	B
3	H	A	**C**	C	C	C
4	I	F	F	F	F	F
5	F	G	G	G	G	G
6	A	I	I	**I**	H	H
7	J	J	J	I	I	I
8	B	H	H	H	J	J
9	J	J	J	J	J	J

p 74, l −9 – −2. 2); this time with $l = 6$, $r = 8$. Split selects the element in position 6 (I) as the splitting element and produces the results shown in the fourth intermediate column of Table 2.1. Quicksort (level 2) calls Quicksort (level 3) with $l = 6$, $r = 6$ (Quicksort (level 3) returns from Step 1), and then with $l = 8$, $r = 8$ (Quicksort (level 3) returns from Step 1). Then (level 2) and (level 1) finish to give the results in the output column of Table 2.1.

p 75 ex. 4. What is the worst-case time for the variation of Quicksort?

p 77, l 18. would look in location of x_i

p 79 l −5. the sequence $\{0, c_0,\ c_0 + c_1,\ c_0 + c_1 + c_2,\ \ldots\}$

p 81, eqs. 77–79.

$$A = n - \sum_{1 \le i \le n} a_i, \qquad 77$$

where $a_i = \sum_{0 \le j < i} p_j$. Now define

$$q_i = \text{Prob(the step is done } i \text{ or more times)} = \sum_{j \ge i} p_j. \qquad 78$$

With this definition $q_i = 1 - a_i$. Thus

$$A = \sum_{1 \le i \le n} q_i. \qquad 79$$

p 81 l −18. If the q_i are easier to compute than the p_i (as they were in Section

p 85 l 17 [the degree of $R(i)$ is less than that of $D(i)$]. For example,

p 88 l 11. The method in this section

p 98 ex. 3.

$$\int_0^\infty e^{-t} t^{x-1}\,dt - \Gamma_m(x) = \int_m^\infty e^{-t} t^{x-1}\,dt + \int_0^m \left(e^{-t} - \left(1 - \frac{t}{m}\right)^m \right) t^{x-1}\,dt$$

p 104, l 11. Suppose, for example, that you need to simplify $\sum_i \binom{n+i}{i} x^i$, where $|x| < 1$.

p 110. The smudges on this page appear to be on the plate. If so, a new plate should be shot.

p 126 l 10. $\omega^{n/2}(\omega^{-1/2} + \omega^{1/2})^n = \omega^{n/2}(2\cos(\pi/3))^n$. Likewise, p 130, ex. 2.

$$\sum_i \binom{m}{i}\binom{m-i}{i+j-m}2^{-2i-j}.$$

p 133, l 2 and eq. 180. In general, the Stirling number $\left[{n \atop k}\right]$ is the sum of all the products of $(n-k)$ different integers taken from 1 to $n-1$; that is,

$$\left[{n \atop k}\right] = \sum_{1\le i_1<i_2<\cdots<i_{n-k}\le n-1} i_1 i_2 \cdots i_{n-k}.$$

p 134, l 3 and eq. 186. In general, the Stirling number $\left\{{n \atop k}\right\}$ is the sum of all the products of $n-k$ integers taken from 1 to k; that is,

$$\left\{{n \atop k}\right\} = \sum_{1\le i_1\le i_2\le\cdots\le i_{n-k}\le k} i_1 i_2 \cdots i_k.$$

p 135, l 12. One of these ways, however, has one part empty.

p 144, eq. 223.

$$\sum_i \binom{r}{i}\binom{s+i}{n}(-1)^i = \sum_{i\ge 0} \frac{s!(s+1)^{\overline{i}}(-r)^{\overline{i}}}{i!n!(s-n)!(s-n+i)^{\overline{i}}}$$

p 145, l −5. Many results on hypergeometric functions can be generalized to *basic hypergeometric functions*, which are obtained

p 146, eq. 243.

$$N() = N + \sum_{1\le i\le k} (-1)^i \sum_{1\le j_1<j_2<\cdots<j_i\le k} N(A_{j_1}A_{j_2}\ldots A_{j_i}).$$

p 146, eq. 244.

$$\frac{N(A_1A_2\ldots A_i)}{N} = \frac{N(A_1)N(A_2)\cdots N(A_i)}{N^i},$$

p 147 eq. 245.

$$\phi(n) = n + \sum_{1\le i\le k} (-1)^i \sum_{1\le j_1<j_2<\cdots<j_i\le k} \frac{n}{p_{j1}p_{j2}\cdots p_{ji}},$$

p 147 eq. 246.

$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right).$$

p 147, l −6. *pigeonhole*

p 150, l 10.

$$\min_{x_0\le x\le x_1}\{g_L(x)\} \le y \le \max_{x_0\le x\le x_1}\{g_U(x)\}.$$

p 150, l −2. where x will be required to be in some range $x_0 \le x \le x_1$ and U (alternately L) is an upper (lower) bound on $f^{(n+1)}(x)/(n+1)!$ in the range of x.

p 151, eq. 15.

$$e^x \ge 1+x \quad \text{for} \quad x\ge 0 \qquad \text{and} \qquad e^x \le \frac{1}{1-x} \quad \text{for} \quad 0\le x<1. \qquad 15$$

p 151, l 11. for some c in the range $0 \le c \le x$. Using Principle 9 (with $n = 0$) gives
p 151, eq. 20

$$0.9039 \le (1 - 1/100)^{10} \le 0.9049.$$

p 153, l 2. $n\sqrt{N}$, testing $1 + (n-1)\sqrt{N}$, …, $n\sqrt{N} - 1$. (Actually, there is no need to test against N because we know the value is less than or equal to N).
p 154, l 7. The trees for $\binom{d}{0}$ and $\binom{d}{d}$ each contain a single node.
p 154, l −15. For testing values up to N we need a tree with at least N leaves.
p 154, l −12. tree has at least N leaves.
p 154, l −10. A binomial tree will have at least N leaves
p 154, l −6. h items by doing at most $\lceil (h!N)^{1/h} \rceil + h - 1$ tests.
p 159, l −19 $a_{m+1} > 0$, we have
p 159, l −8 to −6 $U = a_{m+2} + a_{m+3}r + \cdots = (f(r) - \sum_{0 \le i \le m+1} a_i r^i)/r^{m+2}$, which is finite. This establishes eq. (45) with the C for eq. (23) equal to $a_{m+1}/2$
p 163, eq. 71.

$$\left(1 - \frac{1}{100}\right)^{10} = e^{-1/10}\left[1 + O\left(\frac{1}{1000}\right)\right]$$

p 164, ex. 7. Show that (n is the variable)
p 165, l 4. at x_0 if $\lim_{n\to\infty}[f(x_0) - s_n(x_0)] = 0$.
p 169, l −15. *Backtracking* is a method of organizing a search through
p 173, l 22. The probability that a clause does not contain the literal
p 175 l 18. for another example).
p 175, l −10. Let's use r_k for the probability that $P^{(i)}_{k-1}(\mathit{false}, \ldots, \mathit{false}) = \mathit{true}$.
p 175, −1. only literals for the first k variables and all the literals are positive, where k is
p 176, l 2. that a clause does not have this form.
p 177, l −9. Now eq. (102) is a decreasing function of i [because $-(1-p)^{-i}$
p 179, eq. 118.

$$2^{-\frac{\ln\left(1+\frac{t(-\ln(1-p))}{\ln 2}\right)}{-\ln(1-p)}} = \left(\frac{\ln 2}{\ln 2 + t(-\ln(1-p))}\right)^{\ln 2/(-\ln(1-p))}$$

p 179, eq. 119.

$$N_{i_*} = 2^{2v+1}\left(\frac{\ln 2}{\ln 2 + t(-\ln(1-p))}\right)^{t + \ln 2/(-\ln(1-p))}. \qquad (1)$$

p 183, l −5. sum is smaller than the integral, but it is larger than the integral that is obtained if the curve is shifted right one unit.
p 186, The *Bernoulli numbers* B_m, which occur as coefficients in the Bernoulli polynomials, are defined by the recurrence

$$B_0 = 1, \qquad B_n = \frac{-1}{n+1}\sum_{0\le i\le n-1}\binom{n+1}{i}B_i \quad \text{for } n \ge 1. \qquad 158$$

Except for B_1 all the Bernoulli numbers of odd index are zero. The first few Bernoulli numbers are given in Table 4.1.

Higher-degree approximations can be obtained by integrating eq. (153) by parts. To do this we will need to integrate $B_1(x)$. As the integration by parts proceeds, it will give rise to a sequence of polynomials called *Bernoulli polynomials*. The *Bernoulli polynomial* $B_m(x)$ is defined recursively as $\int mB_{m-1}(x)\,dx$ with the constant of integration equal to B_m. An equivalent definition is

p 186, ex. 2 Show that

$$\sum_{1\le i<n} f(i) = \sum_{1\le i<m} f(i) + \int_m^n f(x)\,dx - \frac{1}{2}(f(n)-f(m)) + \int_m^n B_1(\{x\})f'(x)\,dx.$$

p 188, ex. 1.

EXERCISES

1. Show that $\sum_{1\le i<n} i^{1/2} \ge \frac{2}{3}n^{3/2} - \frac{1}{2}n^{1/2} - \frac{241}{1920} + \frac{1}{24}n^{-1/2} - \frac{1}{1920}n^{-5/2}$ and that $\sum_{1\le i<n} i^{1/2} \le \frac{2}{3}n^{3/2} - \frac{1}{2}n^{1/2} - \frac{1}{8} + \frac{1}{24}n^{-1/2}$. Hint: Use the generalized Euler summation formula with $m=3$.

p 190, l 3. approaches $-\int_1^\infty B_m(\{x\})\,dx/x^{m+1}$.

p 190, eq. 173.

$$-\int_1^\infty \frac{B_m(\{x\})}{x^{m+1}}\,dx - R_{mn} = -\int_n^\infty \frac{B_m(\{x\})}{x^{m+1}}\,dx \qquad 173$$

p 191, ex. 4 and 5. Every lower limit of 0 should be changed to 1 on the summations.

p 192, eq. 185.

$$A = 1 + \frac{k}{2N} - \frac{1}{2(N-k)} + O\left(\frac{k^2}{N^2}\right) + O\left(\frac{N}{k(N-k)^2}\right).$$

p 196, ex. 3

$$-1 + \sum_{1\le j\le n} \frac{1}{2j(2j+1)} \le \sum_{i\ge 1} \frac{(-1)^i}{i} \le -\sum_{1\le j\le n} \frac{1}{2j(2j-1)}.$$

p 206 eq. 42.

$$= 3(3(3T_{n/2^3} + n/2^2) + n/2) + n$$

p 209, l 8. with boundary condition $T_{2^0+2} = 1$.

p 209 eq. 70.

$$T_{2^k+2} = 4\cdot 3^k - 2\cdot 2^k - 1$$

p 210 ex. 6. Solve $T(n^2/2^r) = nT(n) + bn^2$.

(This exercise fits without resetting any additional pages.)

p 216 eq. 102.

$$T_n = a(n+1)\lg(n+1) + \frac{b-3a+c}{2}n + \frac{c-a-b}{2}.$$

p 217 l −19.

The classical algorithm for matrix multiplication (Algorithm 1.9 [modified to save one addition in the inner loop])

p 217

EXERCISES

1. Consider a sequence of $2^k - 1$ numbers with the middle number first, then the smaller numbers, and then the larger numbers. The small part (and also the large part) obey the same rule. For $k = 3$ the binary sequence 100, 001, 011, 010, 110, 101, 111 is such a sequence. Show that the sequence formed by these rules causes the Split algorithm to produce an equal division each time.

2. Show that the sequence of the last exercise results in Split exiting the first time it gets to Step 7.

3. Show that if Split exits the first time it gets to Step 7 and if the time to go around the loop in Step 3 is equal to the time to go around the loop in Step 6, then the time for Quicksort obeys the recurrence $T_n = an + b + T_{n_1} + T_{n_2}$ when the splitting element is such that Split divides the data into sets of size n_1 and n_2 (where $n = n_1 + n_2 + 1$).

Remove the previous exercise 1 and renumber the remaining ones.

The changes on p. 217 result in pages 217–244 being retypeset. Some thought should be given to ways to reduce the number of reset pages.

p 220, l 9. Since 2 is less than 3,

p 223, l −2. The material

p 226 l 2. $F_{n+1} = (1/\sqrt{5})(\phi^{n+1} - \hat{\phi}^{n+1})$.

p 235, eq. 186.

$$= \sum_{0\le n<k} z^n \sum_{0\le i\le n} a_i T_{n-i} + \sum_{n\ge k} b_n z^n.$$

p 236, l 6. By replacing i with $k - i$ the left side of eq. (190) can be rewritten as

p 237, l 7 to −8.

$$\frac{T_p(z)}{(1-\lambda_p z)^{\beta_p}} = (-1)^{\beta_p+1} \sum_{0\le q} \sum_{\max\{0,q-\beta_p+1\}\le i\le q} c_{p,q-i} \binom{-i-1}{\beta_p-1} \lambda_p^i z^q, \qquad (202)$$

so using $d_q(p)$ for the coefficient of z^q in $T_p(z)/(1-\lambda_p z)^{\beta_p}$, we have

$$d_q(p) = (-1)^{\beta_p+1} \sum_{\max\{0,q-\beta_p+1\}\le i\le q} c_{p,q-i} \binom{-i-1}{\beta_p-1} \lambda_p^i \qquad (203)$$

$$= (-1)^{\beta_p+1} \lambda_p^q \sum_{\max\{0,q-\beta_p+1\}\le i\le q} c_{p,q-i} \binom{-i-1}{\beta_p-1} \lambda_p^{-q+i}. \qquad (204)$$

Now replace $i - q$ by i' ($i = q + i'$) and drop the prime to obtain

$$d_q(p) = (-1)^{\beta_p+1} \lambda_p^q \sum_{\max\{-q,-\beta_p+1\}\le i\le 0} c_{p,-i} \binom{-i-q-1}{\beta_p-1} \lambda_p^i. \qquad (205)$$

Finally replace i by $-i$ to obtain

$$d_q(p) = (-1)^{\beta_p+1}\lambda_p^q \sum_{0\le i\le \min\{q,\beta_p-1\}} c_{p,i}\binom{i-q-1}{\beta_p-1}\lambda_p^{-i}. \tag{206}$$

Thus the form of $d_q(p)$ is an exponential in q (i.e., λ_p^q) times a polynomial in q of degree $\beta_p - 1$ [i.e., the rest of the right side of eq. (206)].

To obtain the coefficient of z^q in the generating function $G(z)$ for the homogenous case, it is necessary to sum the contributions from each $T_p(z)$ to obtain

$$\sum_{1\le p\le j} d_q(p). \tag{207}$$

p 239, eq. 219

$$f_{1,0}2^1 + f_{1,1}1\cdot 2^1 = 1,$$

p 239, l −5. so the solution that matches

p 240, ex. 2. Find the general solution of the recurrence $F_{n+2} + F_{n+1} + F_n = n$. Hint: First try to find a particular solution of the form $c_1 n + c_0$.

p 240, ex. 4.

$$T_n = 4T_{n-1} - 5T_{n-2}$$

p 247, l −3. the grand median and the remaining elements. The remaining

p 248, l −2. For eq. (253)

p 249, eq. (255).

$$\beta \ge \tfrac{1}{9}\beta + \alpha + \beta + \tfrac{13}{18}\beta + 4\alpha + 4\beta \tag{255}$$

$$\ge 73\tfrac{1}{3} + 5\tfrac{5}{6}\beta. \tag{256}$$

Solving eq. (256) gives

$$\beta \le -12\tfrac{4}{7}. \tag{257}$$

p 249, l 8. $\beta = -12\frac{4}{7}$.

p 249, l −10 give

p 249, eq. (259).

$$14\tfrac{2}{3}\,n - 7830$$

p 256, ex. 3.

$$J_{n+1}(z) - \frac{2n}{z}J_n(z) + J_{n-1}(z) = 0,$$

p 256, ex. 4. Show that the $F(n) = {}_2F_1(n,(b+a)/(c+1);b;c+1)$ is a solution to the recurrence $ncF_{n+1} + [(1-c)n + a]F_n + (b-n)F_{n-1} = 0$. Find the general solution to the recurrence.

$ncF_{n+1} + [(1-c)n + a]F_n + (b-n)F_{n-1} = 0$. Find the general solution to the recurrence.

p 259, 46.

$$\sum_{0\le i<k+1} r_{i+1}(n)[B(n-i) - B(n-i-1)] = r_0(n)B(n) - r_{k+1}(n)B(n-k-1).$$

p 261, ex. 4. Show that this equation is satisfied by $n^{\overline{r}}$

p 263, eq. 83.

$$A_i = (i-1)!\Big(A_1 + Y_0 \sum_{1\le j<i} \frac{1}{j!}\Big).$$

For large i the sum is close to $e-1$.

p 266. Exercises move to page 268.

p 266, eq. 100.

$$a_0(n)B_j(n) = -\sum_{1\le i\le k} a_i(n)B_j(n-i).$$

p 268. New section called Annihilation of the Nonhomogeneous Part.

These two changes cause the rest of the chapter (pp 266–276) to be reset.

p 272, l 6. with the boundary conditions $p_{1i} = \delta_{0i}$

p 273, eq. 146.

$$G_n(z) = \frac{(-1)^n}{z}\binom{-z}{n}.$$

p 273, eq. 147.

$$G_n(z) = \frac{(-1)^n}{n!z}\sum_i (-1)^{n-i}\begin{bmatrix} n \\ i \end{bmatrix}(-z)^i = \frac{1}{n!}\sum_i \begin{bmatrix} n \\ i \end{bmatrix} z^{i-1},$$

p 273, l 10 The right side of eq. (143) is the product

p 274, l 17. P_1 and P_2

p 275 eq. 165.

$$q_{13}i^3 + q_{12}i^2 + q_{11}i + q_{10} = i(q_{13}(i-1)^3 + q_{12}(i-1)^2 + q_{11}(i-1) + q_{10}) + i^2 - 2i,$$

p 275 eq. 166.

$$\begin{aligned} q_{13}i^3 + q_{12}i^2 + q_{11}i + q_{10} = {} & q_{13}i^4 - 3q_{13}i^3 + 3q_{13}i^2 - q_{13}i + q_{12}i^3 - 2q_{12}i^2 \\ & + q_{12}i + q_{11}i^2 - q_{11}i + q_{10}i + i^2 - 2i. \end{aligned}$$

p 276 eq. 170

$$q_{13} - q_{12} + 2q_{11} - q_{10} = -2$$

p 278, l −10. The boundary condition is $C_0 = 0$.

p 278, eq. 7.

$$C_n = n + 1 + \frac{2}{n}\sum_{0\le i<n} C_i \qquad \text{for } n \ge 1, \tag{7}$$

p 279, eq. 10.

$$\begin{aligned} nC_n - (n-1)C_{n-1} &= n^2 - (n-1)^2 + n - (n-1) \\ &\quad + 2\sum_{0\le i<n} C_i - 2\sum_{0\le i<n-1} C_i && (9) \\ &= 2n + 2C_{n-1} \qquad \text{for } n \ge 2, && (10) \end{aligned}$$

p 279, eq. 12–113. a first order linear equation. The solution for the boundary condition $C_1 = 2$ is

$$C_n = 2 \prod_{2<j\le n} \frac{j+1}{j} + \sum_{2\le i\le n} 2 \prod_{i<j\le n} \frac{j+1}{j} \qquad 12$$

$$= 2 \sum_{1\le i\le n} \frac{n+1}{i+1} = 2(n+1)(H_{n+1}-1). \qquad 13$$

p 279, eq. 14.

$$C_n = 2(n+1)(H_{n+1}-1) = 2n\ln n + O(n). \qquad 14$$

p 287, l 8-9. To simplify this equation, we can use the fact that $C(x)$ obeys eq. (159). We can rewrite eq. (159) as

p 287, l −8. We can apply eq. (37) to the last term of

p 287, l −3 and apply eq. (37). Thus we have

p 287, eq. 54.

$$\sum_{0\le i\le n-1} C_i C_{n-i} = C_{n+1} - C_n.$$

p 288, eq. 55.

$$K_n = \sum_{0\le i\le n-1} 2C_{n-i-1}K_i + C_{n+1} - 2C_n + \delta_{n0}.$$

p , ex. 3. $t_n = n - 1 + 2\sum_{1\le i\le n-1} t_i$

p 295, eq. 90-91

$$= aze^z + 2e^{pz}\Big[a(1-p)ze^{(1-p)z} + 2e^{p(1-p)z}\Big(a(1-p)^2ze^{(1-p)^2z}$$

$$\vdots$$

$$= az \sum_{0\le i<v} 2^i(1-p)^i e^{(1-p)^iz} \prod_{0\le j<i} e^{p(1-p)^jz}$$

$$+ 2^v G_0((1-p)^vz) \prod_{0\le j<v} e^{p(1-p)^jz}$$

p 298, eq. 111.

$$0 = -l_3 + l_4$$

p 298, eq. 113.

$$C_n = l_1 + (l_1-2)n + H_n + 2nH_n. \qquad 113$$

p 298, l −5. we considered the recurrence

p 304, −11. If we say that the height of a tree is the number of nodes on the longest path to the root, then the number of such trees with height no more than n is given by

p 304, −6. The first term in the recurrence allows for the empty tree.

p 308, ex. 4.

$$\phi_n = (1-p)(\phi_1\phi_{n-2} + \phi_2\phi_{n-3} + \cdots + \phi_{n-2}\phi_1) \qquad \text{for } n \ge 2,$$

p 314, ex. 1.

$$f_n(x) = \min_{0 \le y \le x} \{ y \ln y + f_{n-1}(x-y) \}$$

p 315, −9 label any of the leaves in its left subtree.

p 322, eq. 204.

$$t_n = n - 1 + \sum_{1 \le i \le n-1} (t_i + t_{n-i}), \qquad 204$$

p 331, Step 4. If $i > F_{2t+1}$ and $Y_i = \mathit{true}$, then set $M_{i-F_{2t+1}} \leftarrow \mathit{true}$.

p 339, eq. 70

$$1 = c_{11} + c_{12} + c_{13},$$

p 339, eq. 73

$$c_{11} = \frac{2 - \lambda_2 - \lambda_3 + \lambda_2\lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}.$$

p 339, eq. 77

$$c_{11} = \frac{\lambda_1^2}{(3\lambda_1 + 1)(\lambda_1 - 1)}.$$

p 340, eq. 85

$$= \frac{(\lambda+1)\lambda^2}{3\lambda+1}\lambda^n + O(0.738^n).$$

p 340, eq. 91

$$x_3(l) = \frac{\lambda^{l+1}}{(3\lambda+1)(\lambda-1)} + O(0.738^l) \approx 0.336\lambda^l.$$

p 341, l 16. where I is the identity matrix and A is the matrix

p 342, eqs. 104–106.

$$\begin{aligned} T_{\text{Polyphase Merge}} &= \sum_{1 \le j \le n} \frac{L\lambda^2}{(3\lambda+1)(\lambda-1)} + O(0.401^l) \\ &= \frac{nL\lambda^2}{(3\lambda+1)(\lambda-1)} + O(1) \\ &= \frac{\lambda^2}{(3\lambda+1)(\lambda-1)\lg\lambda} L \lg N + O(1) \approx 0.704 L \lg N\,. \end{aligned}$$

p 342, l −14. Merge about 80 percent faster than Simple Merge

p 343. Check with Q. Stout to see if there are any more errors on this page.

p 344, eq. 114.

$$\frac{d\,P_k(t)}{dt} = \mu P_{k-1}(t) - (k\lambda + \mu)P_k(t) + (k+1)\lambda P_{k+1}(t),$$

p 345, eq. 117.

$$A(t,v) = at + 2\sum_i \binom{t}{i} p^i (1-p)^{t-i} A(t-i, v-1).$$

p 346, l 1. (using the boundary conditions of the original problem), which reduces to

p 346, −2. The *general solution* of eq. (126) is the function p 348, eq. 149–151.

$$x'(n,i) = x\left(\frac{sn-qi}{ps-qr}, \frac{-rn+pi}{ps-qr}\right) = x(ci+a[n-i], di+b[n-i]),$$
$$y'(n,i) = y\left(\frac{sn-qi}{ps-qr}, \frac{-rn+pi}{ps-qr}\right) = y(ci+a[n-i], di+b[n-i]),$$
$$z'(n,i) = z\left(\frac{sn-qi}{ps-qr}, \frac{-rn+pi}{ps-qr}\right) = z(ci+a[n-i], di+b[n-i]).$$

p 351, eq. 169.

$$a_{ni} = \binom{\lfloor\frac{n+i}{2}\rfloor}{i} + \binom{\lfloor\frac{n+i-1}{2}\rfloor}{i}.$$

p 355, −12. When $n = 1$, H_{ni} is zero except for $i = 0$, and $H_{10} = 1$, so F obeys

p 356, ex. 8

$$\frac{i}{n}T_{ni} = (n-i)T_{n-1,i-1} + iT_{n-1,i} \quad \text{for } n \geq 3.$$

p 356, ex. 8 The number of k-dimensional cubes (k-cubes) in an n-cube obeys the recurrence

p 360, l 5. The material in this section is adapted from Cohen [81],

[Carefully check that the above correction is made correctly!]

p 370.

Parent	*Class*	*Parent*	*Class*	*Parent*	*Class*	*Parent*	*Class*
3	4,5	11	11,12	21	22,23	29	30,31
5	3,6	14	15,16	23	24,25	32	33,34
6	2,7	17	17,18	26	27,35	36	37,40
8	9,14	18	19,21	28	29,32	37	38,39
9	10,13						

TABLE 9.1. The nontrivial equivalence classes for the flow graph in Figure 9.5. Two nodes are equivalent if there exists a node with arcs to both of them. The equivalence classes with just one node are not given.

p 377, l −17. Input: A text string T which is an array

p 378, l 2.

$next_j$ equals the largest i such that

p 380, l −6. calls to Union and Find.

p 380, eq. 3.

$$nt_0 + t_1 f(A, S)$$

p 387, l −3. This is a much better

p 388, Step 4.

If $y \neq t$, then set $z \leftarrow y$, $y \leftarrow parent(y)$, $parent(z) \leftarrow t$, and repeat this step.

p 389, Fig. 9.13. A dot is needed in the middle of the three long paths. That is in the right path of the third figure, the second from the left path of the fourth figure and the lower part of the rightmost path of the fourth figure.

p 393, TABLE 9.9. 2 $\quad 2^{A_{x-1}}$

p 393, l −2. to evaluating the polynomial $\sum_{0\leq k<n} x_k y^k$ at the points $y_j = \omega^j$ for $0 \leq j < n$.

p 394, l 4 and many other
p 395, l 1. so eq. (37)
p 397, l −2 one for each of the two subpart,
p 398.

Level	*Calculation*
2	$y_0 \leftarrow y_0' + y_1' = x_j + x_{j+4}$
	$y_1 \leftarrow y_0' - y_1' = x_j - x_{j+4}$
1	$y_0 \leftarrow y_0' + y_2' = x_j + x_{j+4} + x_{j+2} + x_{j+6}$
	$y_2 \leftarrow y_0' - y_2' = x_j + x_{j+4} - x_{j+2} - x_{j+6}$
	$y_1 \leftarrow y_1' + \omega^2 y_3' = x_j - x_{j+4} + \omega^2 x_{j+2} - \omega^2 x_{j+6}$
	$y_3 \leftarrow y_1' - \omega^2 y_3' = x_j - x_{j+4} - \omega^2 x_{j+2} + \omega^2 x_{j+6}$
0	$y_0 \leftarrow y_0' + y_4' = x_0 + x_4 + x_2 + x_6$
	$+x_1 + x_5 + x_3 + x_7$
	$y_4 \leftarrow y_0' - y_4' = x_0 + x_4 + x_2 + x_6$
	$-x_1 - x_5 - x_3 - x_7$
	$y_1 \leftarrow y_1' + \omega y_5' = x_0 - x_4 + \omega^2 x_2 - \omega^2 x_6$
	$+\omega x_1 - \omega x_5 + \omega^3 x_3 - \omega^3 x_7$
	$y_5 \leftarrow y_1' - \omega y_5' = x_0 - x_4 + \omega^2 x_2 - \omega^2 x_6$
	$-\omega x_1 + \omega x_5 - \omega^3 x_3 + \omega^3 x_7$
	$y_2 \leftarrow y_2' + \omega^2 y_6' = x_0 + x_4 - x_2 - x_6$
	$+\omega^2 x_1 + \omega^2 x_5 - \omega^2 x_3 - \omega^2 x_7$
	$y_6 \leftarrow y_2' - \omega^2 y_6' = x_0 + x_4 - x_2 - x_6$
	$-\omega^2 x_1 - \omega^2 x_5 + \omega^2 x_3 + \omega^2 x_7$
	$y_3 \leftarrow y_3' + \omega^3 y_7' = x_0 - x_4 - \omega^2 x_2 + \omega^2 x_6$
	$+\omega^3 x_1 - \omega^3 x_5 - \omega^5 x_3 + \omega^5 x_7$
	$y_7 \leftarrow y_3' - \omega^3 y_7' = x_0 - x_4 - \omega^2 x_2 + \omega^2 x_6$
	$-\omega^3 x_1 + \omega^3 x_5 + \omega^5 x_3 - \omega^5 x_7$

p 399, l 2. different ways and then does not make any additional use of the original
p 399, l −12. the variable q used in Step 2 has a leading zero, followed by the j higher-order bits of the i in reversed order, followed by $k - j - 1$ trailing zeros.
p 399, Step 2. For $0 \le i < n$ do the rest of this step. If $i_{k-j-1} = 0$, then set $q \leftarrow 0 i_{k-j} i_{k-j+1} \ldots i_{k-1} 00 \ldots 0$, $odd \leftarrow \omega^q x_{i+2^{k-j-1}}$,
p 401, l −8. We need (see exercise 3.4.1–2)
p 405, l 13. (for special values of i such numbers are called Mersenne primes in the first case and Fermat primes in the second case).
p 420, l 4.

Arrays T_1, T_2, T_3, T_4, and T_5.

p 454 eq. 32

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) dy.$$

p 454 eq. 33

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

p 463 eq. 70.

$$\mathrm{Prob}(X \geq 70) = \left(\frac{1}{2}\right)^{100} \sum_{i \geq 70} \binom{100}{i} \approx 3.98 \times 10^{-5}$$

p 463 eq. 73.

$$\mathrm{Prob}(|X - 50| \geq 20) \leq \frac{25}{400} \approx 0.062 \qquad (2)$$

p 463 l -12. reduce the limit to 0.031.
p 464 eq. 83.

$$\sum_{i \geq x} \binom{n}{i} p^i (1-p)^{n-i} \leq \exp[-ax + n\ln(1 - p + pe^a)]$$

p 478, l-17. -2 rather than $-2a_j$.
p 479 eq. 136-137

$$\sum_{1 \leq i \leq 3} [y_i - a - bx_i] = 0,$$
$$\sum_{1 \leq i \leq 3} [y_i - a - bx_i]x_i = 0.$$

p 479 eq. 138-139

$$\sum_{1 \leq i \leq 3} y_i - a \sum_{1 \leq i \leq 3} 1 - b \sum_{1 \leq i \leq 3} x_i = 0,$$
$$\sum_{1 \leq i \leq 3} y_i x_i - a \sum_{1 \leq i \leq 3} x_i - b \sum_{1 \leq i \leq 3} x_i^2 = 0,$$

p 488, eq. 7

$$\oint_\Gamma \frac{dz}{z - z_0} = \int_0^1 e^{-2\pi i\theta}\, d(e^{2\pi i\theta}) = 2\pi i \int_0^1 d\theta = 2\pi i. \qquad (7)$$

p 494, eq. 25

$$f(x) = x^{m-n}\left(\frac{1}{1 - x_1/x}\right)\left(\frac{1}{1 - x_2/x}\right)\cdots\left(\frac{1}{1 - x_n/x}\right) \qquad (3)$$
$$= x^{m-n} + (x_1 + x_2 + \cdots + x_n)x^{m-n-1} + \cdots, \qquad (25)$$

p 496, ref. 16 Milton Abramowitz and Irene A. Stegun (eds.)
p 501, ref. 109. *Eine neue Art von Zahlen, ihre Eigenschaften und Anwendung in der mathematischen Statistik*
p 505, eq. 2.79

$$A = \sum_{1 \leq i \leq n} q_i.$$

p 531, Delete Euler, number 353.
p 531, Eulerian number 353.