Final Purdom
12-15-2000 B503

1. A person flips three coins, a penny (1 cent), a nickel (5 cents), and a dime (10 cents). The person keeps those coins that land heads-up, and returns the coins that land tails-up.
   a. What is the average value of the coins that the person keeps?
   b. What is the variance in the value of the coins that the person keeps?

2. Suppose you have N baskets where N_ϕ baskets have neither item A nor B, N_A baskets contain item A (without regard to whether or not the baskets have item B), N_B baskets contain item B (without regard to whether or not they have item A), and N_{AB} baskets contain both items A and B.
   a. Suppose N=2, $N_A = 1$, and $N_B = 1$. What are the upper and lower bounds on the values of N_ϕ and N_{AB}?
   b. Suppose N=2, $N_A = 2$, and $N_B = 2$. Give upper and lower bounds on the values of N_ϕ and N_{AB}.
   c. Suppose you are given N, N_A, N_B, and N_ϕ. What is the value of N_ϕ?
   d. Suppose you are given N, N_A, and N_B. Give upper and lower bounds on the values of N_ϕ and N_{AB}.
3. The recurrence $x = 1 + e^{ax}$ has at least two solutions for real $x > 0$ when a is positive and near zero. Find approximations to these solutions.
   a. Hint: a solution with x near 2 can be found by doing a power series expansion of e^{ax}. When a is near zero and x is not large, this is a useful expansion.
   b. Hint: when x is a good bit larger than a, e^{ax} will be large. Asymptotic iteration is useful for approximating this solution.

4. Solve the following two recurrences
   a. $T_{2n} = 3T_n + an + b$.
   b. $T_{2n,2m} = 3T_{n,m} + anm + bn + c$.

The remaining two questions deals computing $C = AB$ where matrix A has size $2n + 1$ by $2p + 1$ and matrix B has size $2p + 1$ by $2m + 1$. Ideally, your final answers to each question should have a form that makes it easy to tell which question's algorithm is faster (at least as judged by the number of multiplications done).

5. Add a row and column of zeros to matrices A and B, so that they have sizes $2n+2$ by $2p+2$ and $2p+2$ by $2m+2$ respectively. Compute C (actually an expanded version of C with size $2n+2$ by $2m+2$) by using one level of Strassen's Algorithm. (In other words, each subproblem generated by Strassen's algorithm is done using the classical algorithm. How many multiplications of matrix elements are done all together? Include the multiplications by the elements known to be zero and the multiplications used to compute those parts of the expanded C that will eventually be discarded. (Omitting the special multiplications would lead to a more interesting algorithm, but it would lead to a more difficult question.)

6. Partition matrices A and B as shown:

	$2p$ columns	1 column		$2m$ columns	1 column
$2n$ rows	A_{11}	A_{12}	$2p$ rows	B_{11}	B_{12}
1 row	A_{21}	A_{22}	1 row	B_{21}	B_{22}

The matrix C has a similar partition, and it is computed as follows: $C_{11} = A_{11}B_{11} + A_{12}B_{21}$, $C_{21} = A_{21}B_{11} + A_{22}B_{21}$, $C_{12} = A_{11}B_{12} + A_{12}B_{22}$, and $C_{22} = A_{21}B_{12} + A_{22}B_{22}$. The product $A_{11}B_{11}$ is done using one level of Strassen's Algorithm followed by the Classical Algorithm. All other multiplications are done using the Classical Algorithm. What is the total number of multiplication of matrix elements done? Please clearly list the amount of work for each subpart of the problem so that part credit can be given even if you make some minor errors.