Final | Purdom
12-16-04 | B403

1. What is the value of $\sum_{1\le i\le 1{,}000{,}000} i$?

2. Suppose you flip two biased coins repeatedly. The i-th coin ($1 \le i \le 2$) is flipped n_i times and it has probability p_i of heads.
   a. What is the probability that this flipping results in a total of i heads?
   b. What is the expected number of heads?

3. Solve the recurrence $x_i = 3x_{i-1} + i$?

4. Find an asymptotic solution to the equation $x = t + \ln x$ for large t.

5. Consider the recurrence $x_i = f(i)x_{i-1} + g(i)x_{i-2}$ where $x_0 = 0$, $x_1 = 1$, and where $f(i)$ and $g(i)$ are both positive.
   a. Is x_i positive, zero, negative, or a quantity whose sign depends on i?
   b. If you have upper and lower bounds on the size of $f(i)$ and $g(i)$ can you give some bounds on the size of x_i? If so give the bounds, if not explain why.

6. Suppose you compute $x_i = (x_{i-1} + n/x_{i-1})/2$ starting with $x_0 = 1$ using rational arithmetic. Then you can write $x_i = a_i/b_i$ where a_i and b_i are integers, but this representation is not unique unless you impose additional conditions (such as requiring that the fraction be in lowest terms). Assume n is an integer just to keep things simple.
   a. Write a pair of recurences for a_i and b_i. (Since we are not requiring lowest terms, there are many possible recurrences. Any correct recurrence is satisfactory, but an easy to solve recurrence with fairly small b_i is better than a more complex recurrence that has bigger b_i.)
   b. Solve the recurrences if you can. If you can not solve the recurrence for general n, but can solve it for $n = 2$ then do that. If you can not solve the recurrences even with $n = 2$, then say so, and say a little about how big the solution is or about why the problem is difficult.
   c. If you notice any common factors between your a_i and b_i, give the common factors. Otherwise say whatever useful you can say about the problem.