Final 5 Questions Purdom
12-11-06 B503

1. Suppose you put k items into random locations in an N-item hash table. What is the probability that i items are put into the first location?

2. You have N bags of marbles, B of the N bags have some blue marbles, and R of the N bags have some red marbles.

   a. Give the best possible upper and lower limits on the number of bags that contain both red and blue marbles.

   a. Give the best possible upper and lower limits on the number of bags that contain neither red nor blue marbles.

3. Approximate $\sum_{1 \leq i \leq n} i^{-4}$.

4. Approximate the solution to the equation $x^5 - x = u$ for x in terms of u for large u.

5. Assume that you have a recursive algorithm whose running time is given by $T_{2^n} = 3T_{2^{n-1}} + 3 \times 2^n$ for $n > k$, and a nonrecursive algorithm whose time is given by $T_n = n^2 + n$. If the recursive algorithm is used for all sizes $n > k$ but the nonrecursive algorithm is used for smaller sizes, compute the following:

   a. Give a closed form formula in terms of n and k for T_{2^n} when $n > k$.

   b. Give the value of k that leads to the smallest time.

   c. Give a closed form formula in terms of n for T_{2^n} when $n > k$ and the best value for k is used.