Final
 12-12-94

Purdom
 C455

1. How many single digit multiplies are needed when multiplying two n digit numbers using Algorithm 1.8

2. Consider $\sum_{1\le n} \left\lfloor \frac{100}{n} \right\rfloor$. This is a finite sum since the summand is zero for large n. Find upper and lower bounds for this sum. For full credit the bounds should be close together.

3. Solve the recurrence $T_n = 3T_{n/2} + n$.

4. Give a formula for T_m when $T_n = 3T_{n/2} + m$. (This is the kind of recurrence one might have when, through poor programming, one had the overhead portion of a recursive program use the original problem size rather than the current problem size.)

5. Consider the clause $C(x, y, z) = l_1(x) \vee l_2(y) \vee l_3(z)$, where each $l_i(w)$ is either the Boolean variable w or the negation of the variable w. Give a short list of functions F_1, F_2, ..., and the function F so that

$$C(x, y, z) = \exists v F$$

where

$$F = F_1(v, w_1) \wedge F_2(v, w_2) \wedge \dots,$$

each w_i is one of

$$x,\ \neg x,\ y,\ \neg y,\ z,\ \text{or}\ \neg z,$$

and where v may take one of *three* possible values.

Your answer, therefore, will consist of truth tables for the F_i and the formula F. If you wish, you may have F of the form

$$F = F_1(v, w_1) \wedge F_2(v, w_2) \wedge \dots,$$

each w_i is one of

$$l_1(x),\ \neg l_1(x),\ l_2(y),\ \neg l_2(y),\ l_3(z),\ \text{or}\ \neg l_3(z).$$

This alternate form for the answer probably leads to a simpler answer.

Hint: when designing your formula it is useful to have v to select which literal in the clause is responsible for causing to clause to evaluate to *true*.