

1. Suppose you roll a red die (which has numbers 1 to 6). If the number rolled is n , you then roll n green dice (each of which has the numbers 1 to 6) and sum the numbers showing on the green dice.
 - a. Give the minimum, maximum, and average value for the sum.
 - b. Give the variance of the sum.

2. Simplify $\sum_{1 \leq i \leq n} i \lfloor \lg i \rfloor$.

3. Define

$$H(v, i, j, k, m, n) = \sum_l \binom{m-1}{l} \binom{n}{l+j-m+1} \binom{n+m-l-j-1}{k} x^l y^j z^k.$$

Simplify $\sum_j \sum_k H(v, i, j, k, m, n)$.

4. Approximate $\sum_{1 \leq i \leq n} i \lg i$. If possible obtain one more term of accuracy than a simple use of problem 2 would give.

5. Give the solution to the recurrence

$$T_n = T_{n-1} + 2T_{n-2}$$

with boundary conditions $T_1 = 3, T_2 = 5$.