Final 	Purdom
12-19-97 	B501

1. Simplify $\sum_{0 \le k \le n} \sin(k/m)$. Hint: eq. 3-157 (second printing) concerning the relation between exponentials and sin function may be helpful for solving this problem. Using this equation and properties of sin and cos under negation of their argument, you can express the sin function in terms of exponentials.

2. You want to walk from the point 0,0 to the point 3,4 in the shortest way. You are in a city, and there is a road at each integer coordinate. You must walk on the road. You notice there are a number of shortest paths. Thus, you can go 0,0 0,1 0,2 0,3 3,1 3,2 3,3 3,4 for a distance of 7, you can go 0,0 1,0, 1,1 1,2 1,3 3,2 3,3 3,4 for a distance of 7, etc. How many shortest paths are there?

3. Simplify $\sum_{0 \le i \le n} i^k$ for the case where k is much less than n. (In other words, a sum with k terms is a simplification.)

4. Simplify $\sum_i \binom{n}{2i}\binom{2i+1}{2i}$.

5. Approximate $\sum_{0 \le i \le n} i^i$ for large n.

6. Approximate $\sum_{1 \le i \le n} \frac{1}{i^2}$ for large n.

7. When applied to numbers with $2n$ digits, Algorithm 5.2 generates two subproblems of size n by n and one of size (at most) $n+1$ by $n+1$. The $n+1$ sized problem is of special form; the leading digit is either zero or one. For that problem, one could use the fact that

$$(a2^n + b)(c2^n + d) = ac2^{2n} + (ad + bc)2^n + bd,$$

leading to an n by n multiply plus some adds. Use analysis to decide whether this alternate idea has merit, has no merit, or whether analysis can not decide easily this question. Is the answer different for large bases (such as 2^{16} and small bases (such as 2)?