Final 12-18-98 | Purdom B401/B501

1. Compute $f(n)$ as a function of n, where $f(n) = f(n-1) + n$.

2. Compute $\sum_i \binom{a}{a-i-b}\binom{i+b}{b}x^i$.

3. Approximate $\sum_{1 \le i \le n} \frac{1}{i^2}$.

4. Simplify the sum $\sum_{1 \le i \le n} F_i x^i$, where F_i is i^{th} Fibonacci number.

5. How often (in the worst case) is Step 3 (the head of the inner loop) of Algorithm 1.12 (Insertion Sort) done when the n numbers being sorted under the condition that the numbers being sorted are required to obey the following condition: The small k numbers come first (in any order with respect to each other), the second smallest k numbers come next (in any order with respect to each other), etc., so that the numbers are in $\lceil n/k \rceil$ groups with all numbers in one group larger than the numbers in the preceding group? You may assume that n/k is an integer. Your answer should be a function of n and k.

6. The simple approach to the project was to program Quick Sort so that it sorted all files above some size k, but called Insertion Sort on any smaller file that was produced. An alternate approach was to program Quick Sort so that it sorted all files above some size k, but did nothing on any smaller file that was produced. Once all the calls to Quick sort were completed, the alternate approach then calls Insertion Sort on the whole file. Compare the time used by the two methods.

7. Let $P(n, k)$ be the probability that you have n coins at time t. If you have n coins at time k, your number of coins at time $k + 1$ is determined by the following process. You flip each of your coins (obtaining heads with probability 1/2, tails with probability 1/2) one time. Each time you get heads, you are given another coin. Thus, at time $k + 1$ you have all of the coins you had at time k plus between 0 and n additional coins obtained by flipping heads. At time 0 you have 1 coin. Write a recurrence equation for $P(n, k)$. Also solve your recurrence for $k = 0$, 1, and 2 (all values of n).

8. Find an approximate solution of x as a function of t that is valid for large t when $x^2 \ln x = t$.