1. Suppose x is selected from the integers in the range 0 to X inclusive and y is selected from the integers in the range 0 to Y inclusive. For each variable is equally likely that any possible integer is selected. What is the probability that $z = k$, where $z = x + y$?

2. Consider the probability $\binom{n}{k} p^k (1-p)^{n-k}$ when $k = np$ (n and k are both integers). Use Stirling's approximation for the factorial function to approximate this probability when both n and k are large. An ideal answer will have just enough accuracy so that there is a nonzero error term for all p in the range $0 < p < 1$. It will also have an upper and lower bound, such as provided by the big Θ notation.

3. Suppose $T_n = T_{n-1} + 2T_{n-2}$. What is the general solution of this equation?

4. Suppose algorithm 1 takes time

$$t_1 n^{\lg 3} + \text{lower order terms}$$

and algorithm 2 takes time

$$t_2 (n-2)^{\lg 3} + \text{lower order terms.}$$

For which values of t_1 is the first algorithm faster (for large n). Justify your answer.

5. Consider the recurrence

$$T(m, n) = T(m - n, n) + T(n, n) + n \qquad \text{for } m \geq n.$$

Solve this recurrence for those values of m where $m = kn$ and k is a positive integer. (Your answer should have $T(n, n)$ in it.)

6. Student 1 programmed the Karatsubu-Ofman algorithm in base 2^{b_1} and his algorithm needed time

$$t_1 n^{\lg 3} + \text{lower order terms}$$

to multiply two n digits numbers. Student 2 programmed the Karatsubu-Ofman algorithm in base 2^{b_2} and his algorithm needed time

$$t_2 n^{\lg 3} + \text{lower order terms.}$$

For each student figure out how many bits per second the student can multiply. For which values of the parameters does student 1 have the faster algorithm for large n (when the programs are multiplying the same number of bits)? The ideal answer for the last part of the question will say that student 1 has the faster program when t_1 is less than some function of t_2, b_1, and b_2.