Final
5-1-02

Purdom
B403

1. Simplify $\sum_i i^2 \binom{n}{i}$.

2. Suppose you program Strassen's Algorithm so that the running time obeys the recurrence

$$T_n = 7T_{n/2} + an^2 \quad \text{nanoseconds} \quad \text{for } n > b,$$

$$T_n = n^3 \quad \text{nanoseconds} \quad \text{for } n \leq b.$$

   a. What value of b (as a function of a), should you use to obtain the best running time?
   b. What is the running time (when the best b is used)?

3. What is the general solution to the recurrence

$$X_i = X_{i-1} + 2X_{i-2}?$$

4. Suppose you have m distinct objects and n distinct names. How many different ways can you give each object a unique name?
   Example: If you have objects 1 and 2 and names a, b, and c, then there are 6 ways ([1 : a, 2 : b], [1 : a, 2 : c], [1 : b, 2 : a], [1 : b, 2 : c], [1 : c, 2 : a], [1 : c, 2 : b]).

5. Write a fast algorithm for the following problem. Your input is an array of characters $C[i]$ for $1 \leq i \leq n$ and an integer k. Your output is an array of Booleans $B[i]$ for $k \leq i \leq n$. You must set $B[i]$ to *true* if the number of 'A's in $C[i-k+1]$ to $C[i]$ is at least $k - 20$, and to *false* otherwise.
   The ideal algorithm for this problem runs in time proportional to n and independent of k.

6. Suppose you want to compute e, the base of natural logarithms to ten significant figures, using only pencil and paper. The book should suggest several ways of doing this. Try to pick one that is reasonably efficient and explain how much work it would take. Note that this question only asks you to explain how to do the calculation and the amount of work you would need to do. It does not ask you to do the work.