Final
5-2-96

Purdom
C455

1. Suppose it takes b units of time to grade a project. Suppose that *after* each project you also count the number of projects remaining to grade, which takes one unit of time per remaining project. How much time is spent for grading n projects, including the time for all the counting?
2. Suppose a student tried to find the break point between the classical algorithm and Strassen's algorithm by varying the trial break point while observing the time used by his version of Strassen's algorithm. He always used input matrices of size 256 by 256. His measurements show the following. The time is near a minimum for trial break points in the range 17 to 32, but there are small irregular variations in the time as the trial break point is varied. The time is somewhat larger when the trial break point is in the range 9 to 16, and also when it is in the range 33 to 64. In each range the time is nearly constant, but with small fluctuations.
   a. Why does the time not vary much in the range 17 to 32?
   b. Why are there small fluctuations?
   c. What can you determine about the break point from this data? Be as precise as you can and explain your reasoning.
3. Simplify $\sum_j \binom{n}{4j} x^j$.
4. Solve the recurrence $T(n) = wT(n-1) + 2w$.
5. Find an asymptotic solution for x when $x^3 = y - x^{-2}$ that is valid for large y. (There are several such solutions; we are interested in the one where x is real.) Your answer should be acturate to two terms plus a big O term. Most of your credit will be determined by whether you get a correct answer, but for full credit you need a brief proof of the correctness of your answer.
6. Suppose you want a function $f(m,n,p)$ for the code

   If $f(m,n,p) > 0$ then Strassen else Classical

   (with appropriate parameters for the subroutine calls to Strassen and Classical). Suppose the running time for your classical algorithm is well approximated by the formula

   $$C(m,n,p) = amnp + bmn$$

   and the time for your Strassen code is well approximated by the formula

   $$S(2m,2n,2p) = 7S(m,n,p) + c(4mp + 4np + 7mn)$$

   for known values of a, b, and c. What function do you recommend for $f(m,n,p)$? There are two considerations for this question. The most important is to choose f so that you have the best possible recurrence. It is also important to write f in a form where it can be quickly evaluated.
7. Suppose the conditions are the same as in problem 6, and in addition when any of the sizes in Strassen's algorithm are odd, the time is the same as it would be if the size were rounded up to the next even number (i.e., the algorithm is doing pad with zero with no time penalty for the padding). Write a single formula for S that works for any combination of even and odd m, n, and p. (Hint: floor and/or ceiling functions may be useful). Find the best f for this problem.