

1. The circumference of a circle is  $2\pi$ . The perimeter of a hexagon inside the circle will be less than  $2\pi$  and the perimeter of a hexagon outside the circle will be greater than  $2\pi$ . What are the best bounds on  $\pi$  that can be obtained from simple use of these ideas?

2. Simplify  $\sum_{1 \leq i \leq n} \sum_{0 \leq j \leq i} \frac{j}{i}$ .

3. The recurrence  $E_n = E_{n-1}^2/2^n$  with the boundary condition  $E_0 = 1$  is nonlinear. However, if you take the logarithm of the equation, you obtain a linear first order equation in  $\log E_n$ .

- a. Show the equation that results from taking the logarithm. (You may choose any useful base for your logarithm.)
- b. Solve for  $E_n$ .

4. Let  $a_{n+1} = (a_n + b_n)/2$  and  $b_{n+1} = \sqrt{a_n b_n}$ . Define  $\epsilon_n = b_n - a_n$ .

a. Express  $b_n$ ,  $a_{n+1}$ , and  $b_{n+1}$  exactly in terms of  $a_n$  and  $\epsilon_n$ .

b. Express  $b_n$ ,  $a_{n+1}$ , and  $b_{n+1}$  exactly in terms of  $a_n$  and with a power series of cubic order in terms of  $\epsilon_n$ , that is the least significant term should have the form  $f(a_n)O(\epsilon^3)$  for  $\epsilon_n$  near 0, where  $f$  is some function of one variable.

5. Assume  $x^2 e^x = t$ . Find an asymptotic solution for  $x$  as a function of  $t$  that is correct for large  $t$ .