Final
 5-4-95

Purdom
 C455

1. How many single element multiplies are needed when multiplying two n by n matrices using Algorithm 1.9.

2. Define
$$P(v,j,k) = \binom{v}{j,k,v-j-k} p^{j+k}(1-p)^{2v-j-k}.$$
Simplify
$$\sum_{j,k} P(v,j,k).$$

3. With Strassen's algorithm for matrix multiplication, something special must be done when the size is an odd number. One way to do this is to decompose the matrix into a matrix with an even size and a matrix of size one. The even size matrices can be multiplied using Strassen's algorithm and the size one matrices can be multiplied using the traditional algorithm. Give an explicit algorithm based on this idea. So that you don't have to do excessive writing, assume that you have an algorithm named S that does Strassen's algorithm for even size problems.

4. Write a equation for the running time of the algorithm for problem 3 when used to multiply an m by n and an n by p matrix (You do not need to solve the equation).

5. One way to handle odd size matrices is to pad out to the next even size. When this is done, the running time (square problems, highest order terms only) is given by
$$T(2n+1) = 7T(n+1) + a(n+1)^2.$$
Suppose the boundary condition is $T(k) = A$. What is the solution to this equation?

6. Find a good approximation to
$$\sum_{1 \le i \le n} \frac{1}{i^2}.$$

7. On simple single-user computers, the time for Strassen's algorithm obeys the recurrence
$$T(2n) = 7T(n) + an^2 + bn + c$$
for some constants a, b and c. What is the solution to this recurrence?