Final Purdom
8-10-90 C455

1. Solve the recurrence $U(t) = ab^t + cU(t-1)$.
2. Approximate $\cos x - 1 - x^2/2$ with accuracy of one term plus a big $O$.
3. You have $n$ tapes, numbered from 0 to $n-1$. Each tape contains one sorted block. Each block has the same number of items. You also have a supply of empty tapes. You wish to produce a tape containing one sorted block by merging tapes 0 to $n-1$. Assume that one unit of time is needed for each item output.
   a. One way you can do this is merging tapes $i$ and $n-1+i$ to produce tape $n+i$, starting with $i=0$ and continuing until all the items have been merged. After the first round, this algorithms merges the tapes one at a time into the previous result. How many time units does this method take?
   b. Another way to do this is by merging tapes $2i$ and $2i+1$ to produce tape $n+i$. This algorithm merges equal size tapes in pairs until the final result is obtained. How many time units does this method take?
   c. Which method is faster? Is the difference significant? Give an informal explanation of the results of this question.
4. Let

$$p(k,i) = \binom{v-i}{k}(2p)^k(1-p)^{2v-i-k},$$

and

$$s(k,m,j,i) = \frac{\binom{k}{j}\binom{v-i-k}{m-j}}{\binom{v-i}{m}2^j}.$$

Simplify

$$\sum_j p(j+k,i)s(j+k,m,j,i).$$

5. Suppose

$$v^n \le 1 + 2\left[1 + O\left(\frac{1}{pv}\right)\right][1 + 2pv(1+p^2)v]^t,$$

where all variables are positive, $n$ is fixed, $v$ goes to infinity, $t$ goes to infinity, $p$ goes to zero, and $pv$ goes to infinity. Convert this inequality into a condition on $p$. (In other words solve for $p$.)