

1. Suppose n is selected to be in the range 0 to $2^k - 1$ inclusive, and each number has the same chance of being selected.
 - 1a. What is the probability that $n = 0$?
 - 1b. As a function of i , what is the probability that n is both greater than or equal to 2^{i-1} and less than 2^i ?
2. This question concerns the number of digits need to hold the sum of two random integers.
 - 2a. Suppose x is an integer in the range 0 to $2^m - 1$ and y is an integer in the range 0 to $2^n - 1$ where $m \geq n$ (both ranges are inclusive). Each integer is selected at random with each possible value being equally likely. What is the probability that $x + y$ is greater than or equal to 2^m ?
 - 2b. Suppose x is an integer in the range 2^{m-1} to $2^m - 1$ and y is an integer in the range 2^{n-1} to $2^n - 1$ where $m \geq n$ (both ranges are inclusive). Each integer is selected at random with each possible value being equally likely. What is the probability that $x + y$ is greater than or equal to 2^m ?
3. Obtain a simple expression for the value of the two expression below. If appropriate, give an approximation that is good for large n .
 - 3a. $\prod_{1 \leq i \leq n} n^i$.
 - 3b. $\prod_{1 \leq i \leq n} i^i$.

For the next two questions consider the following calculation:

$$U_1 = \lfloor U2^{-a} \rfloor.$$

$$U_2 = (U \bmod 2^a).$$

$$V_1 = \lfloor V2^{-b} \rfloor.$$

$$V_2 = (V \bmod 2^b).$$

$$T_1 = U_1 + 2^c U_2.$$

$$T_2 = V_1 + 2^d V_2.$$

$$W = 2^e U_1 V_1 + (2^f T_1 T_2 - 2^g U_1 V_1 - 2^h U_2 V_2) + 2^i U_2 V_2.$$

This is a generalization of eq. 5-86 from the text, where U and V are inputs, while a, \dots, i are parameters.

4. Write a recurrence equation for the time needed to do the calculation of W from U and V . To keep things simple and to promote uniformity among correct answers please do the follow:
 - 4a. Only consider the time needed for the three distinct multiplications that do not involve powers of two.
 - 4b. Use m for the number of bits of U and n for the number of bits of V .
 - 4c. Use $T(m, n)$ for the time needed to multiply an m -bit integer by an n -bit integer.
 - 4d. Use n with a plus superscript (n^+) as a shorthand for the size of a number that could have n or $n + 1$ bits. (Such numbers arise when adding a number with n bits to a number with n or less bits.)
5. Consider values of the parameters that make this be an integer multiplication algorithm.
 - 5a. What values must the parameters have so that this is essentially the same calculation as eq. 5-86?
 - 5b. What values must the parameters have so that W is the product of U and V ? These values are what you need for a correct multiplication algorithm of this type. For full points, you must find all solutions to the problem. Please indicate whether you have found all or just some of the solutions. Also, explain your reasoning.