

1. Simplify  $\sum_{1 \leq i \leq n} \frac{(i+2)!}{i!}$ .
2. Simplify  $\sum_{1 \leq i \leq n} \frac{i!}{(i+2)!}$ .

Let  $n!!$  be the product of alternate numbers from  $n$  down to 1 or 2. Thus, if  $n$  is even, then  $n!!$  is the product of the even integers up to and including  $n$ , and if  $n$  is odd, then  $n!!$  is the product of the odd numbers up to and including  $n$ .

3. Express  $n!!$  exactly using factorial, powers, and elementary arithmetic operations (do not use  $!!$  in the answer).
4. Express  $n!!$  approximately using powers and elementary arithmetic operations (do not use  $!!$  or  $!$  in the answer). Hint: eq. 3-13 may be useful.

Suppose you have  $n$  distinct objects and you build trees in the following way. The (unique) tree in set  $T_3$  is built by connecting the first three objects to a central node. A tree in set  $i$  is built by taking some tree from set  $T_{i-1}$ , selecting some edge of the selected tree, inserting a node in the middle of the edge, and connecting the  $i^{\text{th}}$  object to the new node. The trees in  $T_3$  and  $T_4$  are:

5. In these trees the objects represent nodes of degree 1. The internal nodes all have degree 3. Prove that the internal nodes all have degree 3.
6. The trees in set  $T_i$  all have the same number of edges. Give a formula for this number.
7. How many tree are in set  $T_i$ ?