

The first four questions are about the Linear Search Algorithm (Algorithm 2.1) with two modifications. First, the items in the array X are in sorted order. Second, there are two possible starting points: 1 and j . (The value of j will vary, depending on the problem.) Thus, when searching for q , the initial value of i is 1 when $q < x_j$, and j when $q \geq x_j$. Assume that items are looked up in random order. Assume that each item being looked up is in the table.

1. The value of j is fixed ($1 \leq j \leq n$). What is the average number of positions searched? Your answer should be a function of n and j .
2. Determine the best value for j in problem 2. What is the average number of positions searched when this best j is used? Your answers should be a function of n .
3. The value of j is the place that the previous successful search found its item. Ignore the first search because j is not defined for the first search. What is the average number of positions searched (the number of times that Step 2 of the algorithm is done)? Your answer should be a function of n , the number of items in the table.
4. Which algorithm does the least searching, the algorithm in problem 1 or the algorithm in problem 2 (when the best value for j is used)?

The next four questions concern Insertion Sort (Algorithm 1.12). Assume that the input consists of two parts. First, n of the numbers the integers from 1 to n , and these numbers are in sorted order. Second, k of the numbers are random real numbers in the range 0 to $n+1$. The numbers of the second type are mixed randomly mixed in with the numbers of the first type. Thus, when $n = 2$ and $k = 1$, the following three sequences are equally likely: $r, 1, 2$; $1, r, 2$; and $1, 2, r$, where r is a random number in the range $0 \leq r \leq 3$. Depending on the sequence and the value of r , either 0 or 1 of the numbers will be out of sorted order.

5. What is the probability that the first number is 1?
6. What is the probability that the first number is 1 and the second number is 2?
7. What is the probability that the exactly j of the first i numbers are integers?
8. What is the average number of times that Step 3 of Algorithm 1.12 will be done with this input?