

1. Suppose you role a pair of dice, where each die has faces numbered from 1 to 6. If sum of the numbers is greater than 2, the role counts, otherwise you role a second time and the numbers from the second role count. (You never role a third time.) What is the average of the sum of the numbers from the roles that count.
2. Simplify $\sum_{0 \leq i \leq n} \frac{1}{i^2 - 1}$ or explain why it does not simplify.
3. Simplify $\sum_i \frac{k! x^i y^{k-i}}{i!(n-i)!}$. (The n and k are not misprints.)
4. Prove that $\sum_{0 \leq i \leq n} i^k = O(n^{k+1})$.
5. Write $\Gamma(n + 1/3)$ in a form where the only gamma function with a non-integer argument is $\Gamma(1/3)$.