

Suppose  $A[1..m]$  is an array of  $m$  distinct integers and  $B[1..n]$  is an array of  $n$  distinct integers. The following algorithm finds the indexes of the pairs of elements, one from  $A$  and one from  $B$ , such that the two elements are equal. Give good upper and lower limits on the number of times each step is done.

1. Suppose  $A[1..m]$  is an array of  $m$  distinct integers and  $B[1..n]$  is an array of  $n$  distinct integers. The following algorithm finds the indexes of the pairs of elements, one from  $A$  and one from  $B$ , such that the two elements are equal. Give good upper and lower limits on the number of times each step is done. (After the exam, consider a better algorithm for the problem.)

Algorithm 1.

- a. For  $i := 1$  to  $m$  do the remaining steps.
  - b. For  $j := 1$  to  $n$  do the remaining steps.
  - c. If  $A[i] = B[j]$ , Output the pair  $[i, j]$ .
2. If  $n$  is an integer randomly selected in the range 0 to 110, what is the average and variance of  $n \bmod 11$ ?
  3. Simplify  $\sum_{1 \leq i \leq n} i3^i$ .
  4. If you randomly select one of the  $2^n$  subsets of  $n$  items, what is the expected number of items in the subset, and what is the variance of that number.

Note: “randomly select” (questions 2 and 4) means each possibility has the same probability.