

Give all answers in the simplest form that you can.

1. Simplify $\sum_{1 \leq i \leq n} \sum_{1 \leq j \leq n} ij$.
2. Simplify $\sum_{5 \leq i \leq n} \frac{1}{i^2 - 4}$.
3. Consider a hash table with n positions. Suppose each new item is assigned a random position. For each case below, give the probability that no position in the table has two or more items.
 - a. One item is put into the table.
 - b. Two items are put into the table.
 - c. Three items are put into the table.
 - d. k items are put into the table.
4. You independently generate two random numbers integers in the range 1 to 6, each integer in the range being equally likely to be generated.
 - a. What is the average value of the sum of the two numbers?
 - b. What is the variance of the sum?
5. Consider a hash table with n positions, where you want to put k items into the table without any collisions. Suppose you also have a supply of hash functions. For this problem you will try the first hash function. If it produces no collisions on the k items, you will use it. Otherwise, you will try the second hash function. You will keep trying hash functions until you find one that produces no collisions for any of the k items.
 - a. What condition do you need between k and n to be sure that there exists some hash function that will produce no collisions.
 - b. Let $p_{n,k}$ be the probability that a randomly selected hash function produces no collisions. How likely is it that the process of trying hash functions will stop when the i -th hash function is tried? Assume that the various hash functions are tried in random order with no effort to avoid repeats. In other words, for each possible mapping of k items into n positions, we assume that we are equally likely to choose a hash function that produces that mapping.
 - c. Suppose the process in part b of this question is modified so that no hash function is tried twice. In other words, the hash function tried on the second step is randomly selected from among those hash functions that do not produce the same mapping as the one selected on the first step, etc. In this case, how likely is it that the process of trying hash functions will stop when the i -th hash function is tried.