Test 1 Purdom
10-19-92 C455

1. Simplify the following sums.
   a. $\sum_i \binom{n}{i}$.
   b. $\sum_i \left[{n \atop i} \right]$. Hint: If you sum eq. 185 from chapter 3 over i you obtain a useful recurrence equation.

2. Give partial fraction decompositions for the following:
   a. $\dfrac{1}{(x-1)(x-2)(x-3)}$.
   b. $\dfrac{1}{\prod_{1\le i\le n}(x-i)}$.

3. The following procedure builds each unrooted binary tree with labelled leaves and unlabelled interior points. Each tree is built in only one way. The first three points form a tree with three edges. For each remaining point, select one of the edges that already exists and add an edge from the selected node to the middle of the selected edge. Repeat until all the points are in the tree.
   Note that each step in the loop adds two nodes (the selected one and the one where the edge is bisected) and two edges (one from the new node and a net increase of one from splitting the selected edge into two edges).
   How many different trees (of the type built by the above algorithm) can be built when you start with n points?