Test 1 Purdom
10-20-98 B403, B503

Remember: For full credit you must work the problem correctly and present the answer in a simple form (when a simple form exists).

1. What is the value of $\sum_{10 \le i \le 100} i$?
2. Suppose you have n distinct numbers in random order. What is the probability that the smallest number is first?
3. What is the probability that the numbers in problem 2 (with their random order) happen to be sorted in increasing order?
4. Suppose you roll 2 ordinary 6-faced die to produce two random numbers, each in the range 1 to 6. Consider the difference between the larger and smaller of the two numbers. What is the probability distribution of this difference?
5. Suppose a biased coin is filled n times. On each flip there is a probability p of heads. What is the probability that the number of heads is an even number? Under what conditions is this probability greater than 1/2?
6. Simplify: $\sum_{1 \le i \le n} i \lceil \lg i \rceil$.
7. Simplify: $\sum_{i_1, i_2} \binom{n}{0, i_1, i_2, n - i_1 - i_2} 2^{i_2} 3^{n - i_1 - i_2}$.
8. Write a fast algorithm to generate all Stirling numbers of the second kind, $\left\{ {a \atop b} \right\}$, for $0 \le a \le n$, $0 \le b \le n$.
9. Simplify: $\sum_i i^2 \binom{m}{2i-1} \binom{n}{2i} \binom{2i-1}{p}$.