

1. What is the sum of the odd integers from 1 to 1000?
2. Simplify $\sum_{m \leq i \leq n} \binom{i}{k}$. Assume $m \leq n$.
3. Simplify $\sum_{2 \leq i \leq n} \left(\frac{1}{i^2 - 1} \right)^2$.
4. Simplify $\sum_i i^2 \binom{r}{i} \binom{s}{n-i}$ where n is an integer.
5. (This question counts as two questions.)
 - a. Write a fast program to compute Stirling numbers of the first kind. You may choose which language to write the program in. In particular, compute a table of $[n_i]$ for $0 \leq n \leq m$, $0 \leq i \leq n$.
 - b Analyze the running time of your program as a function of m . Give the most attention to the leading term in the formula for the running time.
6. (This question counts as two questions.) It relates to the version of Quick Sort where the middle of three numbers is selected for the splitting element.
 - a. Suppose you have a list n of distinct numbers, and you select a number at random. What is the probability that the selected number is the i -th largest ($1 \leq i \leq n$)?
 - b. Suppose you randomly select three *different* numbers from the previous list. (The selection is done so that any particular number will be selected at most one time.) What is the probability that the first number is the i -th largest, the second number is the j -th largest, and the third number is the k -th largest?
 - c. In part b, what is the probability that the first number is the i -th largest, the first number is larger than the second number, and the first number is smaller than the third number?
 - d. In part b, what is the probability that the middle-most of the selected numbers is the i -th largest?
 - e. What is the average position of the middle-most number? (You must calculate the average value of the result from part d.)
 - f. What is the variance of the position of the middle-most number?