

1. A program needs to look up items from a table. There are  $n$  different items in the table. The program will use linear search (Algorithm 2.1). The probability that the program will be looking for the  $i^{\text{th}}$  item in the table is proportional to  $1/i$ .
  - a. What is the constant of proportionality? In other words, the probability is  $\alpha/i$ ; what is the value of  $\alpha$ ?
  - b. What is the average number of searches that will be used to look up an item?
  
2. A person has a ‘regular coin’ (which comes up heads with probability  $1/2$ ) or a ‘trick coin’ (which comes up heads with probability  $3/4$ ). The two kinds of coins look the same. The person decides to do the following experiment to decide which kind of coin he has. He will flip the coin 4 times. If it comes up heads 2 or less times, then he will decide he has a regular coin. Otherwise he will decide that he has a trick coin.
  - a. If he has a regular coin, what is the probability that he will make the correct decision?
  - b. If he has a trick coin, what is the probability that he will make the correct decision?
  
3. What is the asymptotic value of  $\sum_{1 \leq i \leq n} \alpha$ , where  $\alpha = \frac{1}{\sum_{1 \leq i \leq n} (1/i)}$ ? The preferred form of the answer is a power series in  $n$  and  $\ln n$ .
  
4. A person has one of two coins, one has probability  $\alpha$  of heads, the other has probability  $\beta$ , where  $\alpha < \beta$ . The two kinds of coins look the same. He would like to tell which coin he has by flipping.
  - a. Give a careful qualitative discussion of how he can solve the problem. Describe the tests he will do, how he will analyze the results, and what kind of errors he will make. For ease of grading, please use the following notation (if it applies to your method):  $N$ , the total number of flips, and  $H$ , the total number of heads. No formulas are expected in this section. We just want a clear explanation of the methods that will be used and names with definitions for any of the parameters that you will need to answer part b of the question.
  - b. Give formulas for the accuracy that can be expected following the methods in part a. If time permits, include formulas that are practical to use when a large number of flips are done.