

1. Simplify $\sum_{0 \leq i \leq n} (ai^2 + bi + c) \binom{n}{i}$.

2. Simplify $\sum_{1 \leq i \leq n} i[i]$.

3. Give upper and lower bounds for $\sum_{1 \leq i \leq n} \frac{1}{i^2}$ that are close for large n .

An ideal solution will have at least one term in the upper and lower bound that are the same and that goes to zero as n goes to infinity.

4. Approximate the value of x that solves the equation $1 - x - \frac{x^2}{v^2} + \frac{x^3}{v^3} = 0$ for large v . Please carry your approximation out to enough terms for all parts of the equation to have an effect on the your solution.

5. Consider the following algorithm, which has a deck of cards in random order. The deck has four cards with the label 1, four with the label 2, etc. up through label 13. The algorithm repeatedly draws cards until the stopping condition is meet. The algorithm stops after the i^{th} card if the card has a label that is greater than $14 - i$. Thus, on the first round it never stops, but if it gets to the $14^{\text{r}m\text{th}}$ round it always stops. What is the probability that the algorithm stops just after round i ?