

Give all answers in the simplest form that you can.

1. Suppose you flip a true coin 11 times.
  - a. What is the expected number of heads?
  - b. What is the most likely number of heads (the number of heads with the highest probability)?
  - c. What is the variance in the number of heads?
  - d. What is the maximum number of heads that might occur?

2. Simplify  $\sum_{1 \leq i} \binom{6}{i} \binom{i}{3} x^i$ .

3. You have a program that is fed problems where the true answer is 'yes' or 'no'. The program can output the answers 'yes' and 'maybe'. When fed a problem where the true answer is 'no', the program always answers 'maybe'. When fed a problem where the true answer is 'yes', the program answers 'yes' half of the time, and 'maybe' half of the time. If it is fed the same problem (with a true answer of 'yes') repeatedly, the successive results from the program are uncorrelated.
  - a. Explain how to use the program so that you can have a 99.9 percent chance of determining that the answer to a problem is 'yes' for a problem where the true answer happens to be 'yes'.
  - b. Using the method you select for part a, what will happen with problems where the true answer happens to be 'maybe'? What is the probability that a problem with the true answer of 'yes' will give the same result as a problem with the true answer of 'no'?

4. Simplify  $\sum_i \binom{2n}{2p+2i-1} \binom{p+i}{p}$ .

5. You wish to compute the following

$$\sum_{\substack{j \\ M[i,j] \text{ defined}}} M[i,j]W[k,j].$$

There are about 500 values for  $j$ . The calculation will be repeated for about 2000 values for  $i$  and 2 values for  $k$ . Consider the following two approaches for doing the calculation.

- a. Store  $M[i,j]$  in a two dimensional array (element  $[i,j]$  stored in position  $i \times \text{size-of}(j) + j$  past the start of the array), with a special value to indicate undefined. The program has a loop that runs through all the values of  $j$ , and it does the multiplication and addition only in those cases where  $M[i,j]$  is defined.

- b. Let  $P[i]$  be a pointer to a one dimensional array where each cell contains two values:  $M[i, j]$  and  $j$ . In this case only the defined values of  $M[i, j]$  are stored, and they are stored in the initial part of the one dimensional array. (Somewhere you also keep the number of elements stored in the one dimensional array.) The program has a loop that runs through all the elements of the one dimensional array, using the  $j$  value from the cell to determine which  $W[k, j]$  is needed.

Suppose that 1/10th of the  $M[i, j]$  values are defined. If high quality code is written for both methods, what will be their relative speed? Explain and justify your answer.