Test 1 — Purdom

2-28-07 — B503 Analysis of Algorithms

Give all answers in the simplest form that you can.

1. Consider the algorithm below, where each x, y, and z is a one digit number in base b, c is a one-digit number in base b, and p is a two-digit number in base b.

   a. How often is the product $x_i y_j$ computed.

   b. Let M be the time to multiply two single digit numbers, A be the time to add a two-digit number and two one-digit numbers, and D be the time to divide a two-digit number by a one-digit number obtaining a one-digit quotient and a one-digit remainder. (The numbers are in base b.) Give a formula for the running time of the algorithm below in terms of these constants, m, and n. Ignore the time not captured by M, A, and D (loop control and data moving time).

Step **1.** For $i \leftarrow 0$ to $n-1$, set $z_i \leftarrow 0$.
Step **2.** For $i \leftarrow 0$ to $m-1$ do:
Step **3.**   Set $c \leftarrow 0$.
Step **4.**   For $j \leftarrow 0$ to $n-1$ do:
Step **5.**   Set $p \leftarrow x_i y_j + z_{i+j} + c$.
Step **6.**   Set $z_{i+j} \leftarrow p \bmod b$ and $c \leftarrow \lfloor p/b \rfloor$.
Step **7.**   End for j.
Step **8.**   Set $z_{i+n} \leftarrow c$.
Step **9.**   End for i.
Step **10.** Stop.

2. Simplify $\sum_i \binom{i}{2p} \binom{2p+1}{i}$, where p is an integer.

3. Suppose coins of type A come up heads with probability p_A and coins of type B come up heads with probability p_B.

   a. Give a formula for $P_A(n,k)$, where $P_A(n,k)$ is the probability that flipping a coin of type A comes up heads k times when it is flipped n times.

   b. Let $P_A(n,k)$ be defined as in part a, and let $P_B(n,k)$ have a corresponding definition. Write the condition for $P_A(n,k) \le P_B(n,k)$ and solve for k as a function of n, p_A, and p_B.

4. Suppose, using hashing with chaining, you put k items into a b bucket hash table.

   a. If the items go into random buckets, how likely is it that bucket 1 will have i_1 items, bucket 2 will have i_2 items, $\ldots$, and bucket n will have i_n items?

   b. How likely is it that each bucket will have at least one item?

5. Consider using the algorithm in problem 1 with base b^k. Also, let Ak be the time to add a two-digit number and two one-digit numbers, and Dk be the time to divide a two-digit number by a one-digit number obtaining a one-digit quotient and a one-digit remainder. The time to multiply two one-digit numbers is now given by the time needed for the algorithm in problem 1 when it is run using base b, $n = k$, and $m = k$.

   a. If this combined algorithm is used to multiply a number with n digits in base b^k (nk-digits in base b) by a number with m digits in base b^k, how much time is used. (Just like problem 1, this analysis is ignoring some of the time used.)

   b. Compute the difference between the answer to part a and the time needed if the algorithm in problem 1 is used directly (using base b). (Assume no time is needed to convert from the base b^k representation that problem 5 uses and the base b representation that problem 1 uses.)