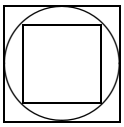
Test 1 Purdom
3-1-04 B503 Analysis of Algorithms

1. Compute $\sum_{0 \leq i \leq 10} 2^i$.
2. One way to get upper and lower limits on π is to draw a unit circle with a regular polygon inside the circle and a regular polygon outside the circle. The perimeter of the inside polygon is less than π, the perimeter of the outside polygon is greater than π. The best bounds for this method are obtained if the polygons just touch the circle. What bounds do you get when the polygon is a square?



3. You have coin A which comes up heads 2/3rds of the time and coin B which comes up heads 1/3rd of the time. Suppose you flip coin A n times and obtain i heads. Also, you flip coin B n times and obtain j heads. For each subproblem, give a general answer in as simple of form as you can find. Also, give a numerical answer for the case that $n = 3$.
   a. What is the probability that $i = j$?
   b. What is the probability that $i < j$?
4. You have sequence of n elements. For each element, the probability that the element is an A is p, independently of the other elements. Thus, the sequence might have any where between 0 and n occurrences of A.
   a. What is the probability that the i element on the list is the *first* A on the list? In other words, what is the probability that the elements in position 1 to $i - 1$ are not A, but the element in position i is A?
   b. What is the probability that A is not in the sequence at all?
   c. Suppose we extend the sequence by having the $n + 1$st element be A. What is the average position of the first A on this extended sequence?
   d. Why is the question: "what is the average position of A on the original list?" not a well formed question?