Give all answers in the simplest form that you can.

1. The following algorithm is supposed to divide the n-digit, base-b, non-negative integer stored in a_1, a_2, ..., a_n by the one-digit, base-b integer stored in d. The result is a quotient whose digits are stored in q and a remainder stored in r. How many times is Step 3 done?

   Step 1. Set $r = 0$.
   Step 2. For i from 1 to n do
   Step 3. Set $q_i = \lfloor (r * b + a_i)/d \rfloor$, $r = r * b + a_i - q_i$.

2. A problem has a solution which is a uniformly selected random number in the range 1 to n. The program in this question solves the problem by testing the numbers 1, 2, 3, ..., n in order; it stops as soon as it solves the problem.
   a. What is the average number of tests done by the program?

   b. What is the variance in number of tests done by the program?

3. The problem for this question is the same as the problem in question 2. The program in this question solves the problem by testing a randomly selected number in the range 1 to n; if that number solves the problem then it stops; otherwise it repeats the process.

a. What is the average number of tests done by the program?

b. What is the variance in number of tests done by the program?

4. A random pair is formed by selecting a first number uniformly from the integers in the range 1 to n and then selecting a second number uniformly from those integers in the range 1 to n that are not the same as the first integer. For each of the following *independent* subproblems, determine the probability that forming *two* random pairs leads to the indicated result.
   a. The first pair is $(1, 2)$. (The second pair has no effect.)

   b. The second pair is the same as the first pair.

   c. One number in the second pair is the same as one of the numbers in the first pair and the other number in the second pair is not the same as either number in the first pair.

   d. Both numbers in the second pair are different from the numbers in the first pair.

5. Simplify $\sum_{i,j} \binom{n}{i} \binom{n-i}{j} x^j$.