

## C241 Homework Assignment 2

1. Exercise 1.2-1 Let  $V = \{a, b, \$\}$ . For each of the following languages  $L_i \subseteq V^+$ , list enough elements to make it clear what each contains.
  - (a) In language  $L_1$  each word has exactly one  $\$$  and equally many  $a$  s as  $b$  s.
  - (b) In each word of language  $L_2$ ,  $a$  s and  $b$  s alternate with any number of  $\$$  s mixed in.
  - (c) In each word of language  $L_3$ , no  $a$  occurs next to a  $b$  .
  - (d)  $L_4 = \{u\$\hat{v} \mid u \in \{a\}^+ \text{ and } v \in \{\$, b\}^+\}$
  - (e)  $L_5 = \{a^k\$\hat{b}^k \mid k \in \mathbb{N}\}$

2. Let  $A = \{a, b\}$ ,  $B = \{a, b, c\}$ , and  $C = \{a, b, c, d\}$ .
- (a) List the language of all words in  $A^+$  that are exactly two letters long. How many are there?
  - (b) List the language of all words in  $B^+$  that are exactly three letters long. How many are there?
  - (c) How many words are there in the language of words in  $C^*$  that are exactly four letters long?
  - (d) How many words are there in the language of words in  $C^*$  that are exactly three letters long?
  - (e) How many words are there in the language of words in  $C^*$  that are exactly three letters long and contain no duplicate letters?
  - (f) How many words are there in the language of words in  $C^*$  that are exactly four letters long and contain no duplicate letters?

3. Let  $P$  stand for the proposition "Sue says it." Let  $Q$  stand for the proposition "Sam saw it." Let  $R$  stand for the proposition "Sid did it." Express the following sentences as formulas involving the logical connectives. If there is more than one way to translate a sentence, use truth tables to explain any differences in the meaning among these translations.
- (a) Sid did it, Sam saw it, and Sue says it.
  - (b) If Sid did it, Sam saw it.
  - (c) Sid did it only if Sam saw it.
  - (d) Sue says it only if Sid did it, and Sam saw it.
  - (e) If Sue says it implies Sam saw it, Sid did it.

4. Rewrite the following English sentences as logical formulas (in symbolic form). Clearly define any propositional constants you use. For example, your answer should start by defining,  $H = \text{"You hate sunlight."}$ , and so forth.
- (a) If you've been bitten by a vampire, then you hate sunlight.
  - (b) If you hate sunlight, then you must have been bitten by a vampire.
  - (c) You hate sunlight if and only if you've been bitten by a vampire.
  - (d) If it's day and it's sunny, or it's night and it's a full moon, I fly my kite.
  - (e) If it's night and it's not a full moon, or it's day and it's not sunny, I eat cake.

5. Write the truth tables for the following logical formulas and state whether each is a tautology, a contradiction, or neither (a contingency).

(a)  $P \wedge (Q \vee R)$

(b)  $(P \wedge \neg P) \Rightarrow Q$

(c)  $P \Rightarrow (Q \vee \neg Q)$

6. Write a logical formula containing propositional variables  $P$ ,  $Q$ , and  $R$  which is:

- (a) a contradiction,
- (b) a tautology,
- (c) true only when  $P$  is true and both  $Q$  and  $R$  are false,
- (d) true exactly when  $P$  is true.

Try to keep your answers as simple as you can and use truth tables to show your answers are correct.

7. Answer the following questions clearly, in your own words.
- (a) What are two methods for proving two propositional formulas are *logically equivalent*?
  - (b) If  $A$  and  $B$  are two logical propositional statements and  $A \mathbf{eq} B$ , then what do we know about the truth tables for  $A$  and  $B$ ?
  - (c) If  $A$  and  $B$  are two logical propositional statements and  $A \mathbf{eq} B$ , then what do we know about the compound statement  $A \Leftrightarrow B$ ?

SUPPLEMENTAL PROBLEM. **The Farmer's Dilema**

A farmer is taking a head of cabbage and a goat to market. He brings along his faithful dog for company. On the road to the market there is a swift river that must be crossed. A small boat is tied to a tree on the bank of the river for this purpose. The boat is so small that it can carry the farmer and *just one* other thing, the cabbage, the goat or the dog. This is a problem: the farmer knows that, if left alone with the cabbage, the goat will eat it. And if left alone with the goat, the dog will eat it. Can the farmer safely get everything across the river? Or does he have to return home and leave the dog behind?