

C241 Homework Assignment 4

1. Given $A = \{a, c, f, g, i, t, w, x\}$,
 - (a) How many permutations are there for these eight letters?
 - (b) How many of these start with the letter t ?
 - (c) How many of these start with the letter t and end with the letter c ?

2. A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if
- (a) There are no restrictions?
 - (b) There must be six men and six women?
 - (c) There must be an even number of women?
 - (d) There must be more women than men?
 - (e) There must be at least eight men?

3. Telephone numbers in the United States and Canada have three groups of digits, *aaa.eee.nnnn* which meet certain conditions:
1. *Area Code*: 3 digits, the first of which is neither 0 nor 1.
 2. *Exchange*: 3 digits, the first of which is neither 0 nor 1.
 3. *Line Number*: 4 digits, with 0000 disallowed.
- (a) How many possible area codes are there?
 - (b) How many possible exchanges are there?
 - (c) How many possible line numbers are there?
 - (d) How many valid 10-digit phone numbers are there?

4. Ms. Johnson's third grade class is putting on the play "Snow White and the Seven Dwarves." There are 7 boys and 9 girls in the class. There are 12 characters in the play:
- Snow White and the Queen (both girls);
 - the King, Hunter and Prince (all boys); and
 - seven dwarves (either gender).
- (a) In Disney's version of the story each dwarf has their own specific name and personality (Sneezy, Dopey, etc...). How many different ways could you cast the play using disney-style dwarves?
- (b) In the original story, the dwarves were indistinguishable from each other. How many different ways could you cast the play using generic dwarves?

5. Use Definition 3.1 to determine *how many students are enrolled in at least one of the classes: C241, C343, C335*. Show your work.
- (a) C335 has 15 students enrolled in it (these students may be enrolled in other classes as well)
 - (b) C343 has 17 students enrolled in it
 - (c) C241 has 20 students enrolled in it
 - (d) 5 students are taking both C343 and C335
 - (e) 7 students are taking both C241 and C343
 - (f) 8 students are taking both C241 and C335
 - (g) 4 students are taking all three classes.

6.

- (a) How many arrangements are there for all the letters in **SOCIOLOGICAL**?
- (b) In how many of those arrangements are **A** and **G** adjacent?
- (c) In how many of the arrangements are all the vowels adjacent?

7. Suppose you want to assign seats for a single row of 4 guys and 4 gals in such a way that each guy is sitting next to *at least* one gal, and *vice versa*. How many ways are there to do this? HINT: Use a decision tree, and practice by solving the 3-guy, 3-gal problem.

8. *Prove:* For all $n, k \in \mathbb{N}$, $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$