C241 Homework Assignment 5

1. Prove that for all natural numbers, n,

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Prove that for all natural numbers, n,

$$\sum_{i=0}^{n} i(i!) = (n+1)! - 1$$

3. Prove that for all natural numbers n, 6 evenly divides $n^3 - n$.

4. Prove that for all natural numbers $n > 4$, $2^n > n^2$.

5. HINT: *Solving this problem requires more than a straightforward algebraic derivation. Think outside the box.*

Prove that for all natural numbers n, $\displaystyle\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$.

6. Recall that the *choose* function, defined

$$\binom{s}{r} = \frac{s!}{r!\,(s-r)!}$$

is the number of different ways to choose r objects from a set of s objects. Prove that for all $n \in \mathbb{N}$,

$$\sum_{i=0}^{n-1} (2i)^2 = \binom{2n}{3}$$

7. For $n \in \mathbb{N}$, let Q, P_1, $P_2, \ldots, P_n$ be propositions. Prove the following:

(a) $Q \wedge \bigvee_{i=1}^{n} P_i \;=\; \bigvee_{i=1}^{n} (Q \wedge P_i)$

(b) $\neg \bigwedge_{i=1}^{n} P_i \;=\; \bigvee_{i=1}^{n} (\neg P_i)$

(c) $Q \vee \bigwedge_{i=1}^{n} P_i \;=\; \bigwedge_{i=1}^{n} (Q \vee P_i)$

(d) $\neg \bigvee_{i=1}^{n} P_i \;=\; \bigwedge_{i=1}^{n} (\neg P_i)$

8. Consider the program

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P:  begin
    {A, B > 0}
    q := 0;
    r := A;
ℓ:  while r ≥ B do
       begin
       q := q + 1;
       r := r − B
       end;
    end {A = qB + r ∧ r < B}
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Use Theorem 4.2 and invariant assertion

$$I \equiv A = qB + r$$

to prove that this program computes the quotient and remainder of A and B.

9. SUPPLEMENTAL PROBLEM COMMENT: *This is neither a calculus question nor a logic puzzle.*

At last! You've gotten out of the dorms and into an apartment. Life can begin. You get moved in and a few days later are looking for a brownie pan. When you open the cabinet door, you see a 12-inch by 12-inch pan, just perfect for your cooking needs. That's the good news.

Unfortunately, there are four cockroaches in the pan, one at each corner. Now you may not have known this, but cockroaches are very amorous creatures, and will always move directly toward the object of their affection. Each of the four in your pan is attracted to the one in the adjacent corner, going counter-clockwise. So all four cockroaches simultaneously start walking toward the one that attracts them. As a result, they start spiraling toward the center of the pan (See the diagram below).

QUESTION: *How far will the cockroaches travel?*