Discrete Structures for Computer Science

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Show that $(p \wedge q) \rightarrow (q \wedge r) \equiv p \rightarrow (q \rightarrow r)$

$\neg(p \wedge q) \vee (q \wedge r)$	Definition of Implication
$\neg p \vee \neg q \vee (q \wedge r)$	DeMorgan's Law
$\neg p \vee ((\neg q \vee q) \wedge (\neg q \vee r))$	Distributive Law
$\neg p \vee (T \wedge (\neg q \vee r))$	Complement Law
$\neg p \vee (\neg q \vee r)$	Identity Law
$\neg p \vee (q \rightarrow r)$	Definition of Implication
$p \rightarrow (q \rightarrow r)$	Definition of Implication

Write an inference proof for the following:

$$\frac{\begin{array}{c} q \wedge \neg r \\ p \vee r \\ (p \wedge q) \rightarrow t \end{array}}{s \vee t}$$

1. $q \wedge \neg r$	premise
2. $\neg r$	(1) Rule of Conjunctive Simplification
3. $p \vee r$	premise
4. p	(2)(3) Rule of Disjunctive Syllogism
5. q	(1) Rule of Conjunctive Simplification
6. $p \wedge q$	(4)(5) Rule of Conjunction
7. $(p \wedge q) \rightarrow t$	premise
8. t	(6)(7) Modus Ponens
9. $s \vee t$	Rule of Disjunctive Amplification

Definitions

C(x) = “x has cheese”
P(x) = “x has pepperoni”
M(x) = “x has mushrooms”
Domain of x: pizzas

I(x, z) = “x was made by z”
Domain of z: Italian chefs

Translate the following predicates into English:

$\exists x : P(x) \wedge M(x)$
$\forall x : P(x) \rightarrow C(x)$
$\forall x \forall z : I(x, z) \rightarrow P(x)$
$\exists z \forall x : I(x, z)$
$\forall x \exists z : I(x, z)$

Translate the following predicates into English:

$\exists x : P(x) \wedge M(x)$	There is a pizza that has pepperoni and mushrooms.
$\forall x : P(x) \rightarrow C(x)$	All pizzas that have pepperoni also have cheese.
$\forall x \forall z : I(x,z) \rightarrow P(x)$	All pizzas that were made by an Italian chef have pepperoni.
$\exists z \forall x : I(x,z)$	There is an Italian chef who made all of the pizzas.
$\forall x \exists z : I(x,z)$	Every pizza was made by an Italian chef.

$\exists x : P(x) \wedge M(x)$

1. Do any pizzas have pepperoni?
2. Do any pizzas have cheese?

$\forall x \forall z : I(x, z) \rightarrow P(x)$

1. Are any pizzas made by Italian chefs?
2. Are all pizzas made by Italian chefs?
3. Do any pizzas have pepperoni?
4. Do all pizzas made by Italian chefs have pepperoni?
5. Do any pizzas have cheese?

$\exists x : P(x) \wedge M(x)$

1. Do any pizzas have pepperoni? Yes
2. Do any pizzas have cheese? Maybe

$\forall x \forall z : I(x, z) \rightarrow P(x)$

1. Are any pizzas made by Italian chefs? Maybe
2. Are all pizzas made by Italian chefs? Maybe
3. Do any pizzas have pepperoni? Maybe
4. Do all pizzas made by Italian chefs have pepperoni? Yes
5. Do any pizzas have cheese? Maybe

You have a collection of 12 movies: 3 comedy, 4 action, 2 horror, and 3 documentaries.
You want your movies grouped by genre. How many ways are there to organize them so that the action movies are first, then the comedies, then documentaries, and finally the horror movies?

- Your friend wants to borrow two of your action movies and a comedy. How many ways are there to choose these movies?
- You're running out of shelf space, so you decide you want to give away two or three movies. How many ways are there to do so and arrange the rest of your movies on a shelf?

- You want your movies grouped by genre. How many ways are there to organize them so that the action movies are first, then the comedies, then documentaries, and finally the horror movies? $4! * 3! * 3! * 2!$
- Your friend wants to borrow two of your action movies and a comedy. How many ways are there to choose these movies? $\frac{4!}{2!(4-2)!} * 3$
- You're running out of shelf space, so you decide you want to give away two or three movies. How many ways are there to do so and arrange the rest of your movies on a shelf? $\frac{12!}{2!(12-2)!} * 10! + \frac{12!}{3!(12-3)!} * 9!$

Show that $n! > 2^n$ for $n \geq 4$.

Base case: $4! > 2^4$

$24 > 16$

Induction hypothesis: Assume $n! > 2^n$, and show that $(n+1)! > 2^{n+1}$.

$(n+1)! = (n+1) * n!$

$2^{n+1} = 2 * 2^n$

We know that $n + 1 > 2$ from the assumption that $n \geq 4$.

We know that $n! > 2^n$ from the induction hypothesis.

Therefore, $(n+1) * n! > 2 * 2^n$, which is the same as:

$(n+1)! > 2^{n+1}$, for $n \geq 4$