C241 Pairwork I: Truth Tables

Done in class on Wednesday, 9/16/09

1) Give truth tables for the following logical statements.

$(p \wedge (p \vee q))$

p	q	p	$\wedge$	$(p \vee q)$
F	F	F	**F**	F
F	T	F	**F**	T
T	F	T	**T**	T
T	T	T	**T**	T

$(p \wedge \neg p)$

p	p	$\wedge$	$\neg p$
F	F	**F**	T
T	T	**F**	F

$(p \vee \neg p)$

p	p	$\vee$	$\neg p$
F	F	**T**	T
T	T	**T**	F

$(\neg p \vee q)$

p	q	$\neg p$	$\vee$	q
F	F	T	**T**	F
F	T	T	**T**	T
T	F	F	**F**	F
T	T	F	**T**	T

$(p \to q)$

p	q	$p \to q$
F	F	**T**
F	T	**T**
T	F	**F**
T	T	**T**

$(p \wedge q) \vee (p \wedge r)$

p	q	r	$(p \wedge q)$	$\vee$	$(p \wedge r)$
F	F	F	F	**F**	F
F	F	T	F	**F**	F
F	T	F	F	**F**	F
F	T	T	F	**F**	F
T	F	F	F	**F**	F
T	F	T	F	**T**	T
T	T	F	T	**T**	F
T	T	T	T	**T**	T

$p \wedge (q \vee r)$

p	q	r	p	$\wedge$	$(q \vee r)$
F	F	F	F	**F**	F
F	F	T	F	**F**	T
F	T	F	F	**F**	T
F	T	T	F	**F**	T
T	F	F	T	**F**	F
T	F	T	T	**T**	T
T	T	F	T	**T**	T
T	T	T	T	**T**	T

2) If a logical statement *always* evaluates to false (in other words, if every line of its truth table is false) then we call it a Contradiction. If a statement *always* evaluates to true (in other words, if every line of its truth table is true) then we call it a Tautology. Which of the above is a contradiction, which is a tautology?

$(p \wedge \neg p)$ is a contradiction, and $(p \vee \neg p)$ is a tautology.

3) If two logical statements have matching truth tables, (so whenever one is true the other's also true, and whenever one is false the other's also false... for all lines of their truth tables) then they'll always behave identically and we say that they're equivalent. We use the symbol $\equiv$ to indicate equivalence, so if P and Q are two equivalent logical statements, then we write $P \equiv Q$. Which of the logical statements above are equivalent?

- Our first statement is actually true exactly when the variable p is true, so we can write: $p \wedge (p \vee q) \equiv p$
- Since the second statement is a contradiction (it will *always* be false) we can write: $p \wedge \neg p \equiv False$ or just $p \wedge \neg p \equiv F$
- Since the third statement is a tautology (it will *always* be true) we can write: $p \vee \neg p \equiv T$
- The fourth and fifth statements evaluate to true at exactly the same lines of their truth tables (and to false at exactly the same lines), so they're equivalent. In english you might phrase this as: "If Bob gains 20 pounds, then Bob will go to the gym" is equivalent to "Either Bob didn't gain 20 pounds, or he went to the gym". And we can write: $\neg p \vee q \equiv p \rightarrow q$.

- And finally, the last two logical statements are also equivalent. This is an example of the 'distribution' law for logic, and it works about like how you'd expect from algebra. We can write: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

Anything else? Answers to your questions. And important pointers!

Important Pointers

- Always write out all your variables on the left of the table (like in the tables above), even if they appear with a negation in the statement. Also, always write out the lines of your truth tables in the *same* order (first FF, then FT, then TF, and so on). It's not really important which order you pick, but pick one and stick with it! Otherwise it's very difficult to compare what's going on between tables.

- Mind your p's and q's! Keep careful track of your variables when you're copying problems, it's easy to confuse a p with a q.

Your Questions

- It's fine to write 0 for False instead of F, and 1 for True instead of T.

- Check out the table logical equivalences on page 75 of your book to see more about the properties of $\wedge$, $\vee$, and $\neg$ (things like $(p \vee q) \equiv (q \vee p)$).

- Another operator we didn't talk about here is "exclusive or". Our $p \vee q$ really means "p is true, or q is true, or they're both true". Exclusive or, which we can write as: $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$ and it means "either p is true, or q is true, but they're never both true". So for instance, if $p =$"it's night", and $q =$"it's day", then the statement $p \oplus q$ would be true.

- Another method of writing truth tables is to build the full statement piece by piece, rather than writing the truth value for each operator directly under the operator. So for the fifth table we could instead write the table below. If you prefer this method, you're welcome to use it:

$(p \wedge q) \vee (p \wedge r)$

p q r	$(p \wedge q)$	$(p \wedge r)$	$(p \wedge q) \vee (p \wedge r)$
F F F	F	F	**F**
F F T	F	F	**F**
F T F	F	F	**F**
F T T	F	F	**F**
T F F	F	F	**F**
T F T	F	T	**T**
T T F	T	F	**T**
T T T	T	T	**T**

- Everyone has their own notation for *and*, *or*, *not*, etc... and you'll probably see many variants in your future. However, just to make the lives of our poor overworked AI and UI simpler, please use the the notation that we use during lecture on your homework: $\wedge$, $\vee$, and $\neg$.