C241 Assignment 10: Graphs

Due Wednesday 04/23/08

1) Let G be a graph with v vertices and e edges. Let M be the maximum degree of the vertices of G, and let m be the minimum degree of the vertices of G. Show that

(a) $2e/v \geq m$

(b) $2e/v \leq M$

2) Draw a graph with the given adjacency matrix

(a)
$$\begin{vmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

(b)
$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

3) Answer the following questions about incidence and adjacency matrices.

(a) What is the sum of the entries in a row of the adjacency matrix for an undirected graph?

(b) What is the sum of the entries in a row of the adjacency matrix for an directed graph?

(c) What is the sum of the entries of a column of the adjacency matrix for an undirected graph?

(d) What is the sum of the entries of a column of the adjacency matrix for an directed graph?

(e) What is the sum of the entries in a row of the incidence matrix for an undirected graph?

(f) What is the sum of the entries in a column of the incidence matrix for an undirected graph?

4) Use the undirected graph G in Figure 1 to answer the questions below:

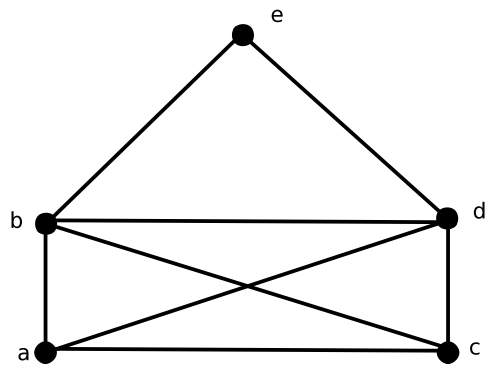

Figure 1: The graph for problem (4).

a) Give an example of a vertex of degree 4 .

b) Give an example of a walk of length greater than 3 from vertex b to d (list the vertices in the order they'd be visited on the walk).

c) Give an example of a cycle starting at a (list the vertices as above).

d) Give an example of a subgraph of graph G (draw it).

e) Give an example of a clique of size 3 (draw it).

f) Give an example of a clique of size 4 (draw it).

g) Why isn't there a clique of size 5?

h) This graph is connected. Which edges could you eliminate to make it so this graph had at least two disconnected components? (draw the resulting graph)

i) Give two examples of spanning trees for this graph (an acyclic subgraph that includes all of the vertices in G) (draw them).

5) Draw five different non-isomorphic trees of size 5 (5 nodes).

6) A complete graph is an undirected graph in which every vertex is directly connected by an edge to every other vertex.

a) Draw a complete graph with 5 vertices.

b) If you could check the degree of a vertex in constant time, how could you check whether a graph with n vertices (and no loops) was complete in $O(|V|)$ time?

c) What's the shortest spanning tree for a complete graph with n vertices?

d) What's the tallest spanning tree for a complete graph with n vertices?

7) Use Dijkstra's algorithm to find the length of the shortest path between the vertices of a and z in the weighted graph in Figure 2. Use the notations and procedure from the textbook!

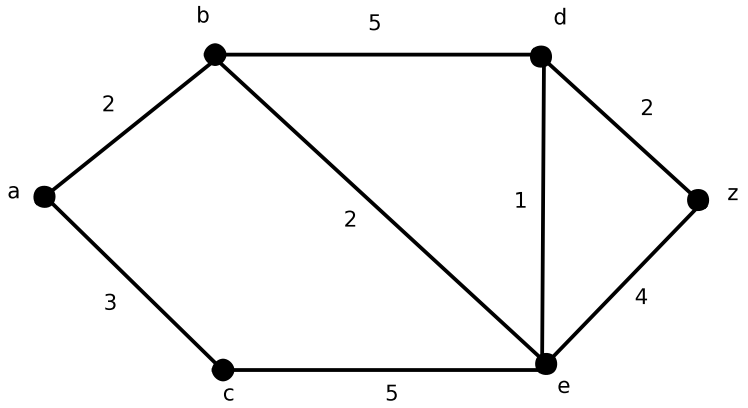

Figure 2: The graph for problem (7).

8) Use (a) depth-first search and then (b) breath-first-search to produce a spanning tree for the graph in Figure 3. Start with node a (i.e., a will be the root node of the spanning tree).

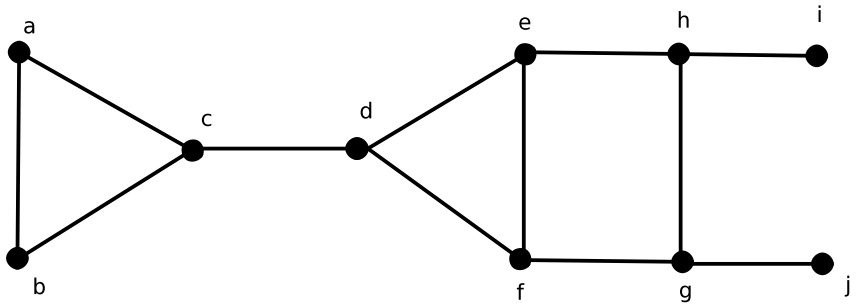

Figure 3: The graph for problem (8).

9) *Bonus:* An Euler Circuit is a cycle which includes every edge in the graph without repeating any edges. Does the graph in Figure 1 have an Euler circuit? If it does have one, draw it. If it doesn't have one, explain why it doesn't. (Hint, what happens if the graph has a vertex with odd degree?)