C241 Assignment 2: Logic Cont'd

Due Wednesday, 1/30/08

1) Simplify the following complex statements using the Algebraic Laws of Logic (along with the equivalence $(p \rightarrow q) \Leftrightarrow (\neg q \vee p)$ which you proved in the last assignment). Label each step you take with the justification for that step.

a) $[(p \vee q) \wedge (p \vee \neg q)] \vee q$

b) $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$

2) Answer the following questions clearly, in your own words:

a) Given two compound statements A and B, describe two different methods for proving that A logically implies B (ie $A \Rightarrow B$).

b) If A and B are two compound statements and $A \Rightarrow B$, then what do we know about the compound statement $(A \rightarrow B)$?

c) Give an example of two compound statements A and B such that A logically implies B, but A and B are not logically equivalent (ie: $A \Rightarrow B$ but $A \not\Leftrightarrow B$). Justify your choice using truth tables.

d) Can you give an example of an A and B such that A and B are logically equivalent, but A does not logically imply B? Why or why not?

3) Label each step of the logical proof below with the justification for that step. Each of your labels should be one of the following: "hypothesis" (or "premise"), one of the logical equivalence laws from the chart at the end of this assignment sheet (Look carefully! There are a couple new laws for $(P \rightarrow Q)$), or a logical inference rule from the other chart at the end of this assignment sheet. Make sure you include the proof lines you're referencing in your label; the label for line (4) has been filled in as an example.

$$
\begin{array}{l}
(\neg p \vee q) \rightarrow r \\
r \rightarrow (s \vee t) \\
\neg s \wedge \neg u \\
\underline{\neg u \rightarrow \neg t} \\
\therefore p
\end{array}
$$

1)	$\neg s \wedge \neg u$	
2)	$\neg u$	
3)	$\neg u \rightarrow \neg t$	
4)	$\neg t$	lines (2)(3) and Modus Ponens
5)	$\neg s$	
6)	$\neg s \wedge \neg t$	
7)	$r \rightarrow (s \vee t)$	
8)	$\neg(s \vee t) \rightarrow \neg r$	
9)	$(\neg s \wedge \neg t) \rightarrow \neg r$	
10)	$\neg r$	
11)	$(\neg p \vee q) \rightarrow r$	
12)	$\neg r \rightarrow \neg(\neg p \vee q)$	
13)	$\neg r \rightarrow (p \wedge \neg q)$	
14)	$p \wedge \neg q$	
15)	$\therefore p$	

4) Why was it acceptable to use logical equivalence laws in the proof above?

5) Do the following logical proofs on your own, using the same style as the proof in problem 2.

a)

$p \rightarrow q$
$\neg q$
$\neg r$

$\therefore \neg(p \vee r)$

b) Do this as a proof by Resolution. First change each of the hypotheses to an 'or' statement, then apply the Rule of Proof by Resolution and simplify until you've completed the proof.

$\neg(\neg p \wedge \neg q)$
$p \rightarrow r$
$(\neg r)$

$\therefore q$

c) Do this as a proof by Contradiction. Add $\neg(S \vee T)$ to the list of hypotheses (i.e., assume the conclusion is false) and then continue the proof until you can derive 'False' as a conclusion (which shows that if $(S \vee T)$ *wasn't* true, then something False would be true, which is a contradiction).

p
$p \rightarrow q$
$s \vee r$
$r \rightarrow \neg q$

$\therefore s \vee t$

d)

$p \rightarrow q$
$\neg r \vee s$
$p \vee r$
$r \rightarrow \neg q$

$\therefore \neg q \rightarrow s$

6) Answer the following questions clearly, in your own words:

a) What is an open statement?

b) Write an open statement with two variables, and list the domain of each variable. For example, I could write: o(x,y) = "x is older than y", where the domain of both x and y is "cities".

c) Describe what the values of x and y need to be for your statement from part (b) to evaluate to true. Give an example of x,y values for which your statement evaluates to true. Give an example of x,y values for which your statement evaluates to false.

d) What are the existential and universal quantifiers? (Give the symbols and english phrases for each).

e) Use a quantifier to bind the variable x in your statement from part b). For what values of y does this new statement evaluate to true?

f) Use a quantifier to bind the remaining free variable in your statement from part e). When does this final statement evaluate to true? What can be said about a statement with no free variables?

A Chart of the Algebraic Laws of Logic

(See page 24 in your textbook)

Note: The symbols P,Q,R,K below can represent complex logical statements.

Identity Laws	$(P \vee \mathbf{F})$	$\equiv$	P		
	$(P \wedge \mathbf{T})$	$\equiv$	P		
Domination Laws	$(P \vee \mathbf{T})$	$\equiv$	$\mathbf{T}$		
	$(P \wedge \mathbf{F})$	$\equiv$	$\mathbf{F}$		
Idempotent Laws	$(P \vee P)$	$\equiv$	P		
	$(P \wedge P)$	$\equiv$	P		
Double Negation	$\neg(\neg P)$	$\equiv$	P		
Commutative Laws	$(P \vee Q)$	$\equiv$	$(Q \vee P)$		
	$(P \wedge Q)$	$\equiv$	$(Q \wedge P)$		
Associative Laws	$(P \vee (Q \vee R))$	$\equiv$	$((P \vee Q) \vee R)$	$\equiv$	$(P \vee Q \vee R)$
	$(P \wedge (Q \wedge R))$	$\equiv$	$((P \wedge Q) \wedge R)$	$\equiv$	$(P \wedge Q \wedge R)$
Distributive Laws	$P \vee (K \wedge Q)$	$\equiv$	$(P \vee K) \wedge (P \vee Q)$		
	$P \wedge (K \vee Q)$	$\equiv$	$(P \wedge K) \vee (P \wedge Q)$		
DeMorgan's Laws	$\neg(P \vee Q)$	$\equiv$	$(\neg P \wedge \neg Q)$		
	$\neg(P \wedge Q)$	$\equiv$	$(\neg P \vee \neg Q)$		
Absorption Laws	$P \vee (P \wedge Q)$	$\equiv$	P		
	$P \wedge (P \vee Q)$	$\equiv$	P		
Negation Laws	$P \vee \neg P$	$\equiv$	$\mathbf{T}$		
	$P \wedge \neg P$	$\equiv$	$\mathbf{F}$		

A Chart of the Inference Rules of Logic

See page 58 in your textbook. *Note: The symbols P,Q,R,S below can represent complex logical statements.*

Modus Ponens	P $\underline{P \to Q}$ $\therefore Q$	**Hypothetical Syllogism**	$P \to Q$ $\underline{Q \to R}$ $\therefore P \to R$
Modus Tollens	$\neg Q$ $\underline{P \to Q}$ $\therefore \neg P$	**Conjunction**	P $\underline{Q}$ $\therefore P \wedge Q$
Disjunctive Syllogism	$P \vee Q$ $\underline{\neg P}$ $\therefore Q$	**Contradiction**	$\underline{\neg P \to \mathbf{F}}$ $\therefore P$
Simplification	$\underline{P \wedge Q}$ $\therefore P$	**Addition**	$\underline{P}$ $\therefore P \vee Q$
Conditional Proof	$P \wedge Q$ $\underline{P \to (Q \to R)}$ $\therefore R$	**Resolution**	$P \vee Q$ $\underline{\neg P \vee R}$ $\therefore (Q \vee R)$
Constructive Dilemma	$P \to Q$ $R \to S$ $\underline{P \vee R}$ $\therefore Q \vee S$	**Destructive Dilemma**	$P \to Q$ $R \to S$ $\underline{\neg Q \vee \neg S}$ $\therefore \neg P \vee \neg R$