C241 Homework 7: Structural Induction

Due Wednesday, 3/26/08

1) Using only the Algebraic Laws of Logic and Induction prove the following:

a) DeMorgans for n variables: $\neg(p_1 \wedge p_2 \wedge p_3 \wedge ... \wedge p_n) = (\neg p_1 \vee \neg p_2 \vee \neg p_3 \vee ... \vee \neg p_n)$.

b) Distributive property for n variables: $p \wedge (q_1 \vee q_2 \vee q_3 \vee ... \vee q_n) = (p \wedge q_1) \vee (p \wedge q_2) \vee (p \wedge q_3) \vee ... \vee (p \wedge q_n)$

2) Use the language P defined below to answer the following questions.

	$P \subset V^+$ with $V = \{(,)\}$
1.	$() \in P$
2a.	$u \in P \Rightarrow (u) \in P$
2b.	$u, v \in P \Rightarrow (uv) \in P$
3.	There is nothing else in P.

a) Which of these strings are in the language P?

(i)	((( )))	(iv)	(( )( ))
(ii)	( ) ( )	(v)	(( ) ( ) ( ))
(iii)	((( ) ( )) ( ))	(vi)	((( ) ( )))

b) Using structural induction, prove that every element in P has the same number of ('s as it has)'s. It may help to use variables like L_u and R_u to represent how many left and right parentheses there are in string u.

3) Below is the definition for the language L and two recursive functions on it. Use these definitions to answer the following questions.

	$L \subset V^+$ with $V = \{a, b, \bullet\}$
1.	$\bullet \in L$
2a.	$u \in L \Rightarrow au \in L$
2b.	$u \in L \Rightarrow bu \in L$
3.	There is nothing else in L.

	$I : L \to L$
1.	$I(\bullet) = \bullet$
2a.	$I(au) = bI(u)$
2b.	$I(bu) = aI(u)$

	$D : L \to L$
1.	$D(\bullet) = \bullet$
2a.	$D(au) = aaD(u)$
2b.	$D(bu) = bbD(u)$

a) Find the strings output by the following functions. *Write out each step of the recursion.*

(i) $I(aabb\bullet)$

(ii) $D(bba\bullet)$

b) Using structural induction prove that for every string $u \in L$, $I(I(u)) = u$.

c) Using structural induction prove that for every string $u \in L$, $D(u)$ **contains an even number of** a**'s (of course, 0 is considered an even number).**

4) A binary tree is a tree where each node either has 0 children (a leaf) or exactly 2 children. Prove that all binary trees have an odd number of nodes.