C241 Assignment 9: Relations and Partial Orderings

Due Wednesday 04/16/08

1) For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

(a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

(b) $\{(2, 4), (4, 2)\}$

(c) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

(d) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

2) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

(a) $x \neq y$.

(b) $xy \geq 1$.

(c) $x = y + 1$ or $x = y - 1$.

(d) x is a multiple of y.

(e) x and y are both negative or both non-negative.

(f) $x = y^2$.

(g) $x \geq y^2$.

3) Let R be a relation that is reflexive and transitive. Prove that $R^n = R$ for all positive integers n.

4) List the ordered pairs in the relations on $\{1, 2, 3, 4\}$ corresponding to these (incidence) matrices (where the rows and columns correspond to the integers listed in increasing order).

(a) $\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{vmatrix}$

5) Draw the directed graph that represents the relation $\{(a,a),(a,b),(b,c)(c,b),(c,d),(d,a),(d,b)\}$**.**

6) Let R be the relation represented by the matrix $\mathbf{M}_R = \begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{vmatrix}$

Find the matrices that represent

(a) R^2

(b) R^3

7) Which of these relations on $\{0,1,2,3\}$ are quivalence relations? Determine the properties of an equivalence relation that the others lack. For those relations that are equivalence relations, what are the equivalence classes?

(a) $\{(0,0),(1,1),(2,2),(3,3)\}$

(b) $\{(0,0),(0,2),(2,0),(2,2),(2,3),(3,2),(3,3)\}$

(c) $\{(0,0),(1,1),(1,2),(2,1),(2,2),(3,3)\}$

(d) $\{(0,0),(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

(e) $\{(0,0),(0,1)(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)\}$

8) Show that propositional equivalence is an equivalence relation on the set of all compound propositions.

9) Show that the relation R on the set of all bit strings such that $s\ R\ t$ (or, in different notation, $(s,t) \in R$) if and only if s and t contain the same number of 1s is an equivalence relation.

10) Which of these are posets?

(a) $(\mathbf{Z}, =)$

(b) $(\mathbf{Z}, \neq)$

(c) $(\mathbf{Z}, \geq)$

11) Find the lexicographic ordering of these strings of lowercase English letters:

(a) quack, quick, quicksilver, quicksand, quacking

(b) open, opener, opera, operand, opened

(c) zoo, zero, zoom, zoology, zoological

12) Draw the Hasse diagram for the "less than or equal to" relation on $\{0, 2, 5, 10, 11, 15\}$.

13) Draw the Hasse diagram for inclusion on the set $\mathcal{P}(S)$ (the powerset of S) where $S = \{a, b, c, d\}$.