

Ph. D. Qualifying Exam  
Algorithm Design and Analysis portion  
Computer Science Department  
Indiana University  
Fall 2002

This question involves planning a long trip where supplies for the trip must be hauled in. Fuel is the only supply of concern.

You have a truck with a 20 gallon fuel tank. The truck can haul two 10 gallon fuel cans. The truck uses one gallon of fuel for each 10 miles. The starting point is the only place where you can buy fuel.

For the questions below, efficient answers are much better than inefficient answers. Correct answers are much better than no answer or than incorrect answers. So that your answers can be more simple, it is okay to buy extra fuel, but avoid extra travel. Also, assume that when you use a fuel can, you must put all of the fuel from the fuel can into the fuel tank.

1. For each distance up to 1000 miles, give a detailed plan. Here are plans that work for up to 500 miles (you should judge how efficient they are):
  - a. For up to 400 miles: Fill the tank and cans with fuel and just drive to the destination (putting the fuel from the cans into the tank at 200 miles).
  - b. For distances above 400 miles and up to 500 miles. Drive 100 miles towards the destination with one can of fuel. Leave the one can of fuel at the 100 mile point and drive back to the starting point. Fill the tank and cans with fuel. Drive to the 100 mile point and fill up with the fuel that was stored there. Drive to the destination (refilling with the hauled cans at 300 miles).
2. Give a plan (perhaps recursive) that works for any large distance.
3. Compute how much travel is required by your plan.
4. Redo the previous questions so that once you get to your destination you can return to the starting point. The preferred answer to this question involves converting the answers for a one way problem into answers for a related two way problem.