

Visualizing Quaternions

Course Notes for SIGGRAPH 2001

Course Organizer

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Abstract

This tutorial focuses on establishing an intuitive visual understanding of the relationship between ordinary 3D rotations and their quaternion representations. We begin building this intuition by showing how quaternion-like properties appear and can be exploited even in 2D space. Quaternions are then introduced in several alternative representations that do not necessarily require abstract mathematical constructs for their visualization. We then proceed to develop visualizations of quaternion applications such as moving frames and orientation splines. Finally, we briefly discuss the problem of generalizing quaternion concepts to higher dimensions using Clifford algebras.

Presenter's Biography

Andrew J. Hanson is a professor of computer science at Indiana University, and has regularly taught courses in computer graphics, computer vision, and scientific visualization. He received a BA in chemistry and physics from Harvard College in 1966 and a PhD in theoretical physics from MIT in 1971. Before coming to Indiana University, he did research in theoretical physics at the Institute for Advanced Study, Stanford, and Berkeley, and then in computer vision at the SRI Artificial Intelligence Center in Menlo Park, CA. He has published in IEEE Computer, CG&A, TVCG, ACM Computing Surveys, and has over a dozen papers in the IEEE Visualization Proceedings. He has also contributed three articles to the Graphics Gems series dealing with user interfaces for rotations and with techniques of N-dimensional geometry. Previous experience with conference tutorials includes a Siggraph '98 tutorial on N-dimensional graphics, a Visualization '98 course on Clifford Algebras and Quaternions, and a tutorial on Visualizing Quaternions presented at both Siggraph '99 and Siggraph 2000. Major research interests include scientific visualization, machine vision, computer graphics, perception, and the design of interactive user interfaces for virtual reality and visualization applications. Particular visualization applications currently being studied include an astrophysical treatment of the local galactic neighborhood of the sun, the exploitation of constrained navigation for visualization environments, and applications of graphics in dimensions greater than three to mathematics and theoretical physics.

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General Information on the Tutorial

Course Syllabus

Summary: This tutorial will deal with visualizable representations of quaternions, their features, technology, folklore, and applications. The introduction will focus on visually understanding quaternions themselves by exploiting parallels to complex variables and 2D rotations. Starting from this basis, the tutorial will proceed to give visualizations of advanced quaternion applications.

Prerequisites: Participants should be comfortable with and have an appreciation for conventional mathematical methods of 3D computer graphics and geometry used in graphics transformations and rendering. The material will be of most interest to those wishing to deepen their intuitive understanding of quaternion-based animation, moving coordinate frames, and 3D curves and surfaces appearing in graphics and scientific visualization applications.

Objectives: Participants will learn the basic facts relating quaternions to ordinary 3D rotations, as well as methods for examining the properties of quaternion constructions using interactive visualization methods. A variety of applications, including quaternion splines and moving coordinate frames for curves and surfaces, will be examined in this context. Finally, a few facts about the deeper relationship between quaternions and Clifford algebras in higher dimensions will be presented.

Outline: This is a two-hour tutorial and the material will be arranged approximately as follows:

- I. *(45 min)* **Introduction to Rotation Representations.** Develop formulas and techniques for seeing how 2D rotations, orientation frames, and their time evolution equations can be visualized and studied using ordinary complex variables. Develop the parallel relationship between 3D rotations and quaternions.
- II. *(45 min)* **Visualization Techniques for Quaternions.** Visualizing static and moving quaternion frames, along with quaternion splines, as 4D geometric objects.
- III. *(15 min)* **Applications of Quaternion Visualization.** Extend this intuition into the quaternion representation of moving orientation frames.
- IV. *(15 min)* **Clifford Algebras: the Bigger Picture.** Start to see how it all fits into Clifford algebras.

1 Overview

Practitioners of computer graphics and animation frequently represent 3D rotations using the quaternion formalism, a mathematical tool that originated with William Rowan Hamilton in the 19th century, and is now an essential part of modern analysis, group theory, differential geometry, and even quantum physics. Quaternions are in many ways very simple, and yet there are enormous subtleties to address in the process of fully understanding and exploiting their properties. The purpose of this Tutorial is to construct an intuitive bridge between our intuitions about 2D and 3D rotations and the quaternion representation.

The Tutorial will begin with an introduction to rotations in 2D, which will be found to have surprising richness, and will proceed to the construction of the relation between 3D rotations and quaternions. Quaternion visualization methods of various sorts will be introduced, followed by some applications of the quaternion frame representation to problems of interest by graphicists and visualization scientists. Finally, we will briefly touch on the relationship between Clifford algebras and quaternion rotation representations. An extensive bibliography of related literature is included, as well as several relevant reprints and technical reports, a Mathematica implementation of the Quaternion Frenet Equations, and the Meshview software system for viewing 4D objects.

2 Fundamentals of Quaternions

We will begin with a basic introduction to rotations in general, showing how 2D rotations contain the seeds for what we need to understand about 3D rotations; see, for example, [39]. We will then proceed to look at a variety of methods for understanding quaternions and making meaningful pictures of constructs involving them. These methods will range from some of the ideas introduced by Hart, Francis, and Kauffman [53] for motivating the need for double-valued parameterizations of rotations, to theoretical background given in [46, 47, 40, 50].

Traditional treatments of quaternions range from the original works of Hamilton and Tait [35, 81] to a variety of recent studies such as those of Altmann, Pletincks, Juttler, and Kuipers [2, 69, 59, 63]. The 4D frames of the quaternions themselves, in contrast to the relationship between 3D frames and quaternions, are treated in the German literature, e.g., [12, 65].

In our treatment, we will focus on the use of 2D rotations as a rich but algebraically simple proving ground in which we can see many of the key features of quaternion geometry in a very manageable context. The relationship between 3D rotations and quaternions is then introduced as a natural extension of the 2D systems.

3 Visualizing Quaternion Geometry

In order to clearly understand our options for making graphical visualizations of quaternions, we next look at the ways in which points on spheres can be viewed in reduced dimensions, discovering luckily that 3D graphics is *just* sufficient to make a usable interactive workstation system for looking at quaternions, quaternion curves, and even quaternion surfaces. The basic “trick” involves

the observation that if we have a four-vector quaternion $q = (q_0, \mathbf{q})$ obeying $q \cdot q = 1$, then the four-vector lies on the three-sphere S^3 and has only three independent components: if we display just \mathbf{q} , we can in principle *infer* the value of $q_0 = \sqrt{1 - \mathbf{q} \cdot \mathbf{q}}$. We supply a viewer, the Meshview system [51, 52] developed by the presenter and his students, which allows the input and interactive examination of quaternion objects.

4 Quaternion Frames

In this section, we study the nature of quaternions as representations of frames in 3D. Our visualizations again exploit the fact that quaternions are points on the three-sphere embedded in 4D; the three-sphere (S^3) is analogous to an ordinary ball or two-sphere (S^2) embedded in 3D, except that the three-sphere is a solid object instead of a surface. To manipulate, display, and visualize rotations in 3D, we may convert 3D rotations to 4D quaternion points and treat the entire problem in the framework of 4D geometry. The methods in this section follow closely techniques introduced in Hanson and Ma [46, 47] for representing families of coordinate frames on curves in 3D as curves in the 4D quaternion space. The extensions to coordinate frames on surfaces and the corresponding induced surfaces in quaternion space are studied in [40, 50].

The same methods extend to the study of quaternion animation splines, introduced to the graphics community originally by Shoemake [73]. We give an overview of the issues of constructing splines with various desirable continuity properties following the method of Schlag [71] applied to quaternion Bezier, Catmull-Rom, and uniform B-splines. These methods do not easily permit long-lasting periodic motion; this problem, as well as some problems with derivative computation, are resolved by the exponential map approach of Kim, et al. (see, e.g., [61, 32]); The exponential map and its visualization properties are therefore briefly noted as well. Alternative approaches such as the rational quaternion spline method of Jüttler [59, 60] and the variational methods of Barr et al. [10, 70] are mentioned but not treated in detail.

5 Clifford Algebras

The quaternion-based formalism for handling and visualizing rotations works well in dimensions 2, 3, and 4 because in those dimensions the Spin group, the double covering of the orthogonal group, has simple topology and geometry. Going beyond four dimensions is of course much harder. Clifford algebras form the basis used in pure mathematics to treat the Spin groups in arbitrary dimensions (see, e.g., [3, 58]); furthermore, viewed in the context of arbitrary dimensions, studying the Clifford algebra approach provides additional depth to our understanding of dimensions 2, 3, and 4 — we can get a better feeling for what properties are accidents of the low dimension and which are in fact general and extensible concepts.

6 *MeshView* and Mathematica Demonstration Software

The course CDROM should contain a copy of the MeshView 4D visualization package [51, 52], which runs under SGI-IRIX, SUN-SOLARIS, and, with some effort, Linux. This contains a sample of quaternion data sets, as well as many other 4D objects, and has facilities specifically designed for quaternion visualization experiments. There is also a Mathematica notebook `qfrmint.nb` that explicitly implements a numerical integration of the Frenet frame equations in quaternion form, vastly improving the exactly equivalent calculation for the standard Frenet equations implemented by Gray [33].

Acknowledgments

The slightly edited versions of the author's papers from Graphics Gems IV and V [38, 39] are included in the CDROM and Course Notes with the kind permission of Academic Press. Republished in the Course Notes are two key papers from IEEE Transactions on Visualization and Computer Graphics [47], and from the Proceedings of IEEE Visualization [40] of the IEEE Computer Society Press. We thank Ji-Ping Sha for his patience with Clifford algebras, and John Hart for supplying the paper by Hart, Francis, and Kauffman included here.

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