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Bitmaps for a Digital Theory of Everything

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POLYTOPICS

Introduction

How does nature compute?

"*So I have often made the hypothesis that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like the chequer board with all its apparent complexities*" (Richard Feynman)

Could Physics be explained by geometrical and graphical shapes instead of formulas and mathematical statements ?
Could Numbers (quaternions...) be implemented by a simple machinery ?
How many dimensions would Feynman's checkerboard have ?

This talk is not yet a demonstration of this hypothesis, but is right in this way.

Overview

- **1) "Ultimately physics will not require a mathematical statement"**
  1.1) Bitmaps
  1.2) Tritmaps

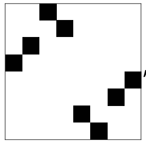
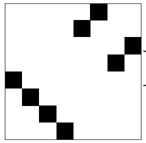
- **2) "In the end the machinery will be revealed"**
  2.1) Machinery components: e8 roots
  2.2) Machinery geometry: 24-cell

- **3) "The laws will turn out to be simple, like the chequer board"**
  3.1) Checker board
  3.2) Rules

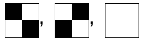
How does nature compute?

1) Ultimately physics will not require a mathematical statement
1.1) Bitmaps

- **Definition : A bitmap is a rectangular array of bits.**
- **Representation : It can be displayed as an array of black or white squares.**
- **Samples : Here are two fundamentals 8x8 bitmaps, named yin and yang :**

{yin = , yang = }

- **Dimension reduction :**

These matrices have only 2 x2 block components of 3 types : 

When we multiply or add them a new componet can appear : 

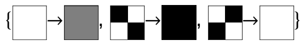
Matrix product and sum could be, bitwise **`BitAnd`** and bitwise **`BitOr`**, but that does not give a useful structure.

We can not use regular matirx product and sum because the result is generally not a bitmap but a matrix of integers {0 to n} where n is the size of the bitmap.

So we need a normalization mecanism to get a bitmap from the result.

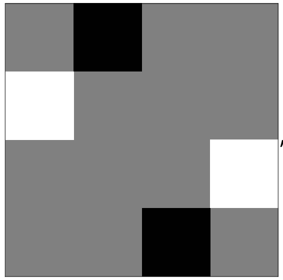
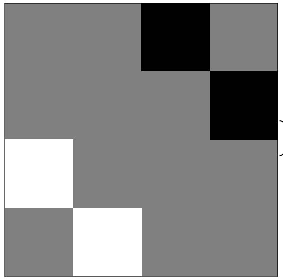
One was found which is "threshold to the max, then quotient by equivalence relation of 2x2 blocks  and .

It is equivalent to a simpler approach using trivalued matrices...

In this approach we will project them into new objects, easier to play with, doing this substitution: 

1.2) tritmaps

- **Definition : A tritmap is a rectangular array of ternary units (trits, e.g. numbers valued in -1,0,1 - thanks to Edward Fredkin, who, after discussion, found the right word "trits").**
- **Representation : It can be displayed as an array of black (1), gray (0) or white (-1) dots.**
- **Samples : Here are the two fundamentals 4x4 tritmaps, named yin and yang :**

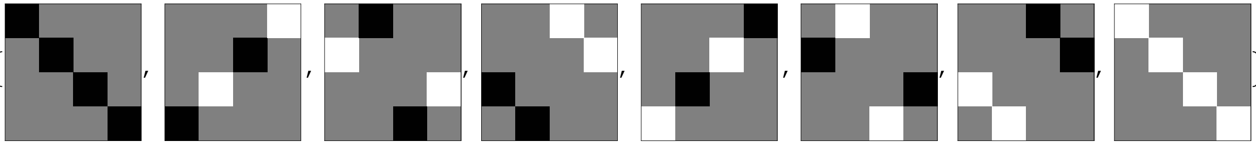
{yin = , yang = }

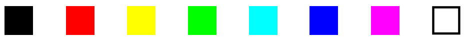
1.2.1) tritmap product

- **Matrix product of tritmaps**

Our generators verifies : $\mathbf{yin}^4 = \mathbf{yang}^4 = \mathbf{1}$, $\mathbf{yin}^2 = \mathbf{yang}^2 = -\mathbf{1}$, $\mathbf{yang.yin} = -\,\mathbf{yin.yang} = \mathbf{yin}^3\mathbf{.yang}$ and $\mathbf{yang}^3\mathbf{.yin} = -\mathbf{yang.yin} = \mathbf{yin.yang}$
So they are multiplicative generators of a 8 elements multiplicative group.

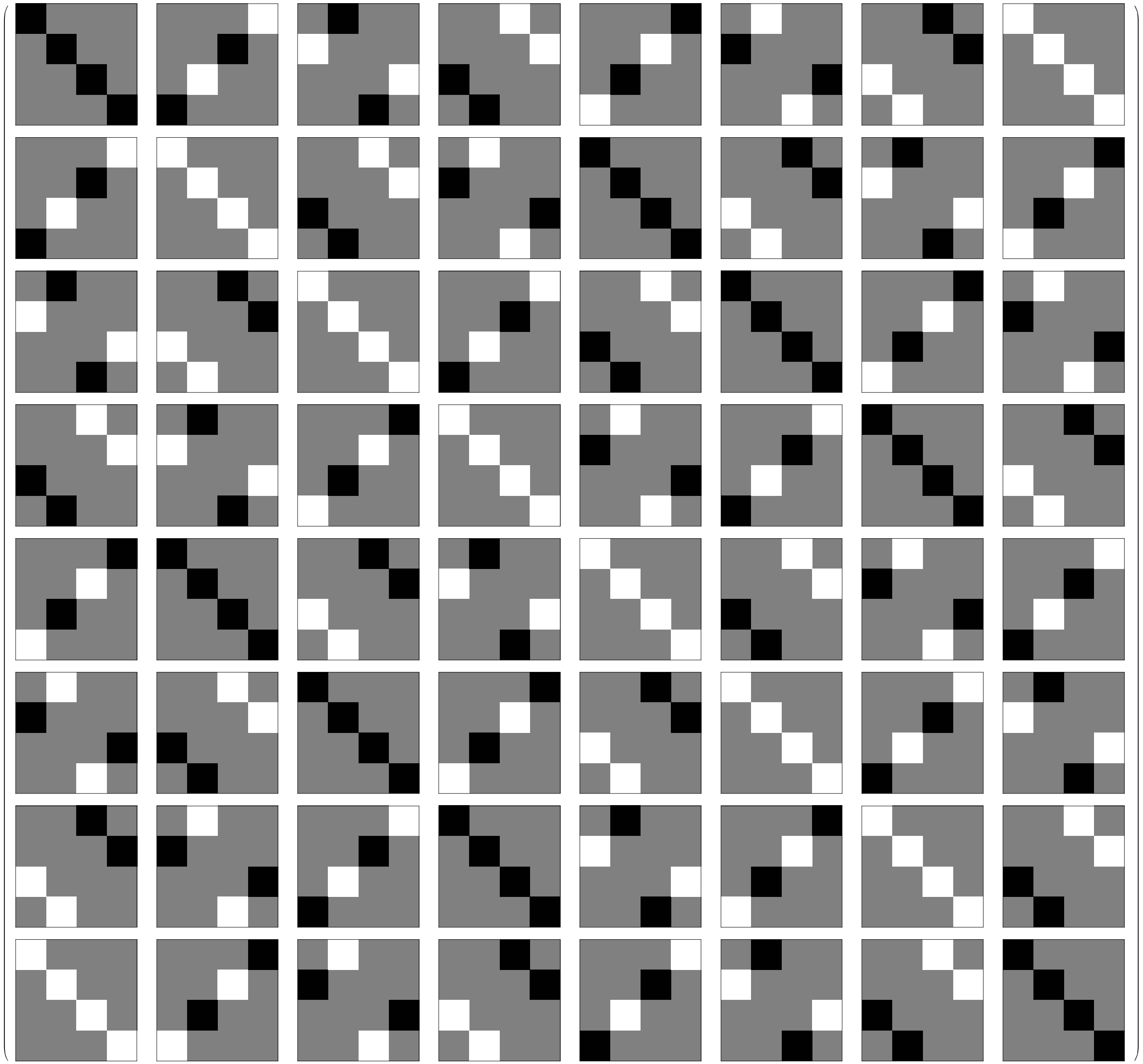
yin^4 yang.yin yin yang^3 yin.yang yin^3 yang yin^2

group8 = {}

$= \{\mathbb{1}\ -\mathbb{k}\ \ \mathbb{i}\ -\mathbb{j}\ \ \mathbb{k}\ -\mathbb{i}\ \ \mathbb{j}\ -\mathbb{1}\} =$ {}

Where upper lines of 4 x4 tritmaps is the dots sequence {K, Y, M, C} on basis $\{\mathbf{yin}^4, \mathbf{yin}, \mathbf{yang}, \mathbf{yin.yang}\}$

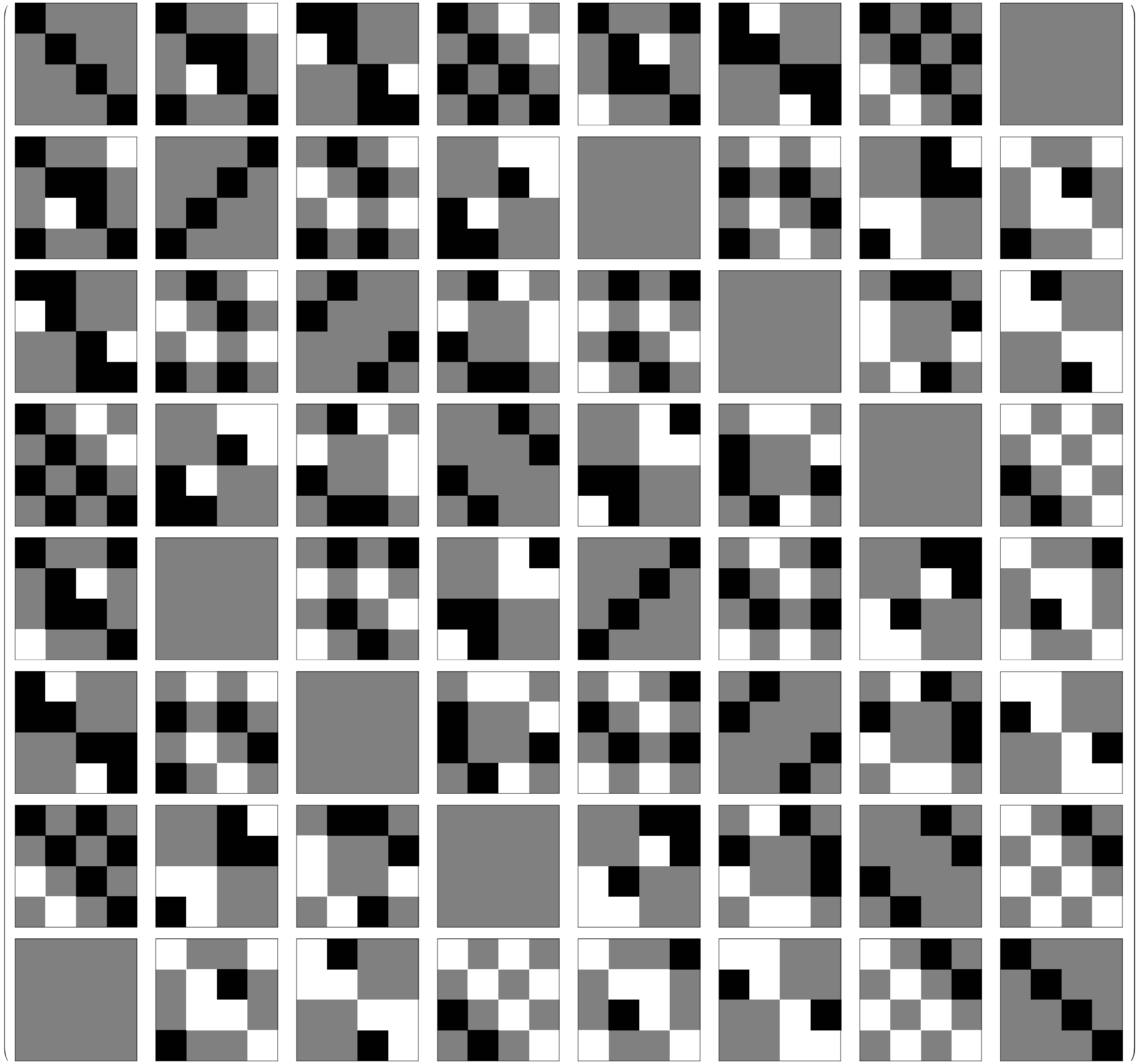
Multiplicative group of eight elements



1.2.2) tritmap sum

- **Sum on the group**

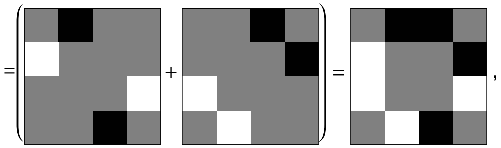
Addition is simply bitwise. Its not a group.

Addition table: 

1.2.3) Triality

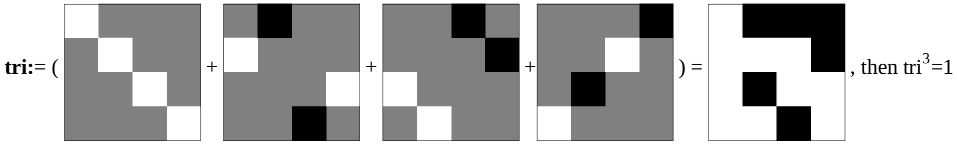
Addition of two elements of the multiplicative group **group8** generates 24 elements which are 3 families of eight products of the group8 by respectively,

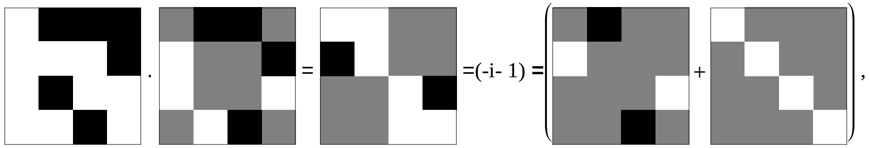
Pick one, for example: **yin** + **yang** = (i + j) =

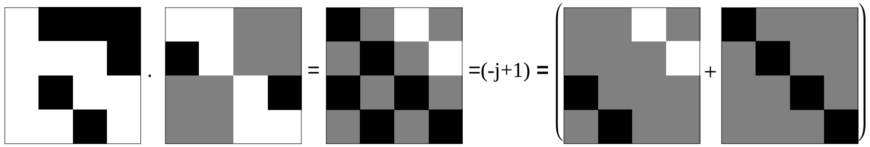


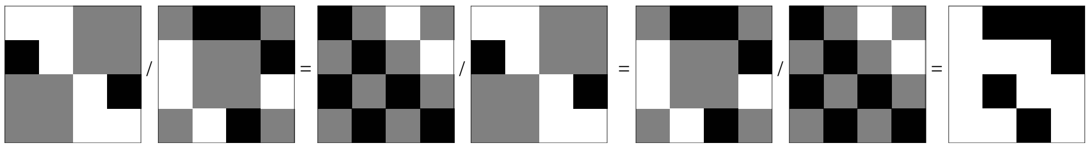
Multiply it by group8, to get 8 of 24 new elements from addition table.

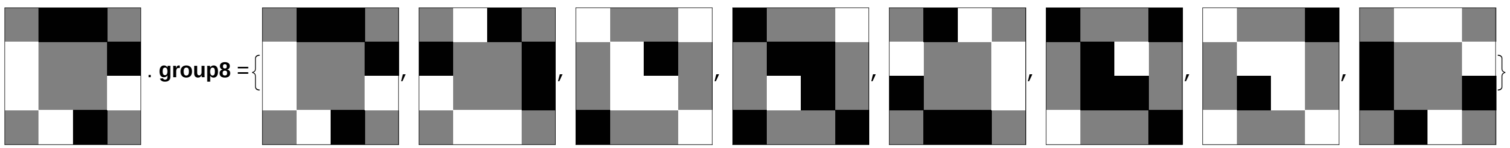
Pick one of remaining 16, for example: **yin.yin.yin**+**yin.yin** = (-i-1)
Get its quotient by yin+yang, its product by yinyinyin+yinyinyang, name it tri

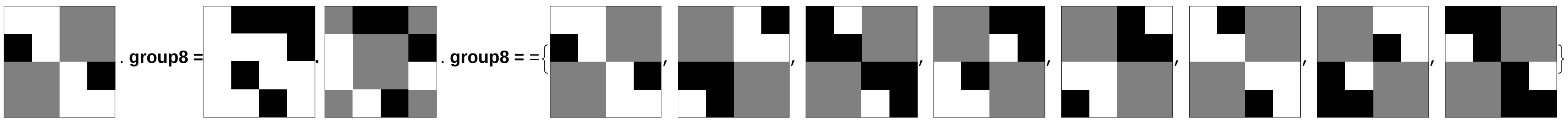


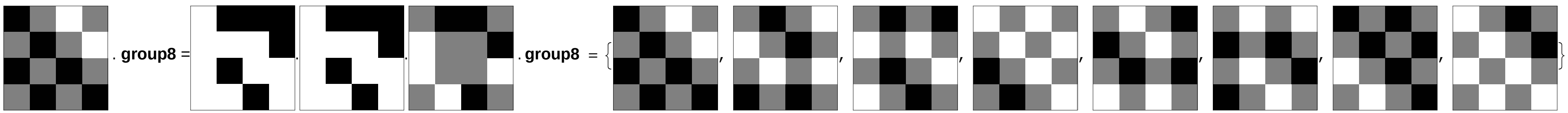




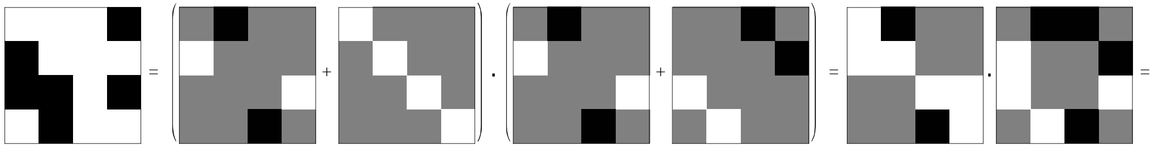




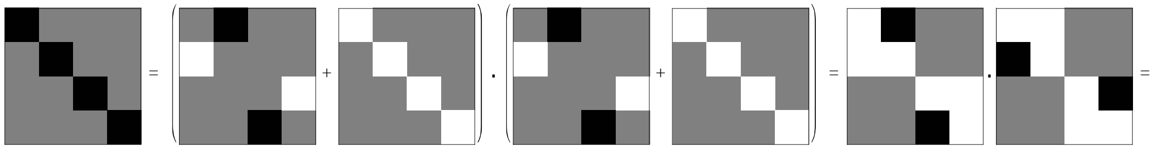




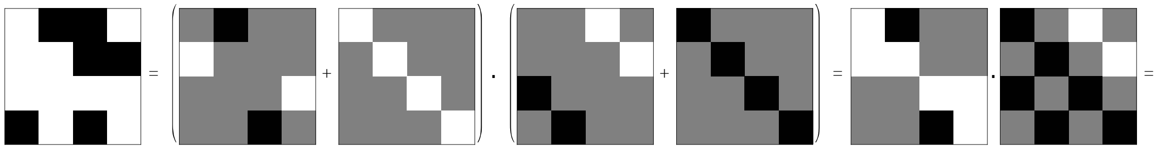
Duality



$= (\mathbb{i} - \mathbb{1})\,(\mathbb{i} + \mathbb{j})$



$= (\mathbb{i} - \mathbb{1})\,(-\mathbb{i} - \mathbb{1})$



$= (\mathbb{i} - \mathbb{1})\,(-\mathbb{j} + \mathbb{1})$

Normative operations

We recognize in the group dyipyiyigroup8 our original group8, because yipyiyigroup8 is by construction the dual of group8, and the dual of the dual is the group itself. But they are not identical as matrix because they now have not only -1,0,1 but also -2 and 2. We have to normalize the matrix product to get a tritmap product.
We do this in one simple step : taking the **Sign**.

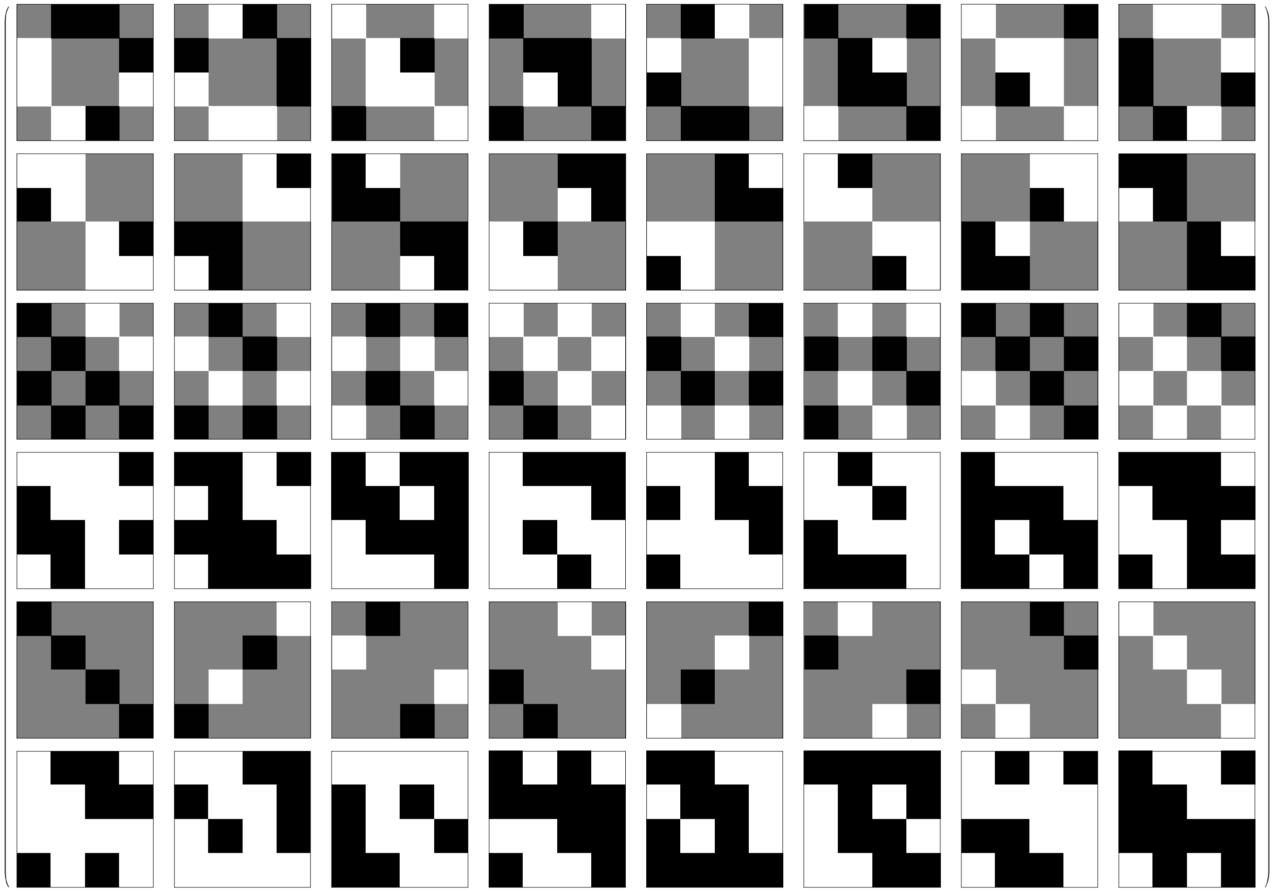
- **tritmaps Operations :**

$A,B \in M_n(\{-1,0,1\})$

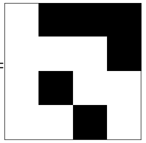
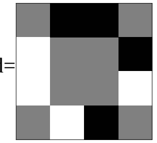
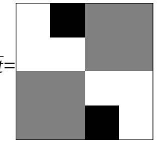
Product: $A \otimes B := \mathtt{Sign}[A \,.\, B]$

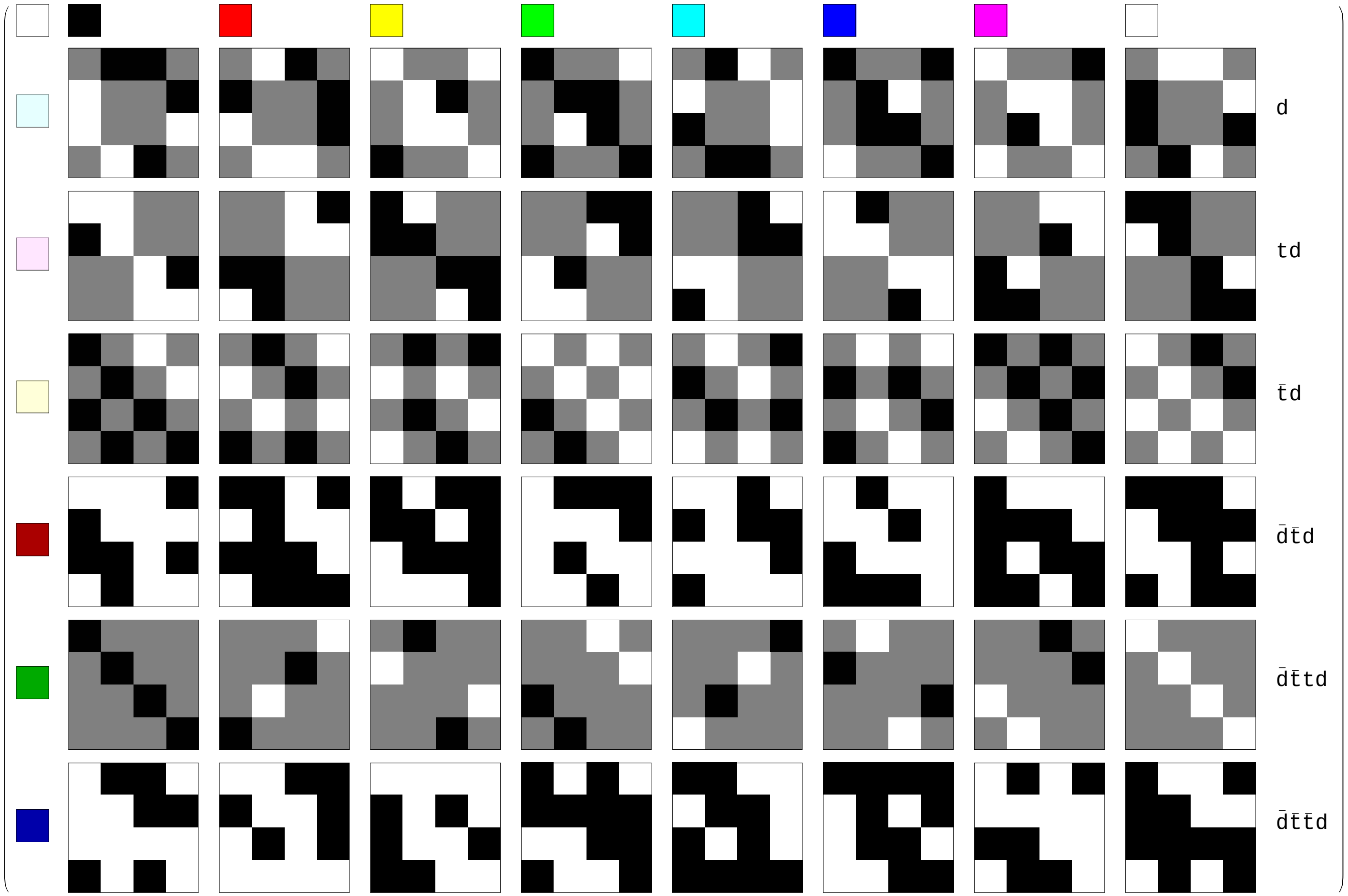
Sum: $A \oplus B := \mathtt{Sign}[A + B]$

- **tritmaps Multiplicative "Half-Bosons" Group :**



"Half-Bosons" Group identification, and Representation using a flavor/color couple:

t=  d=  $\bar{t}=t^2$ $\bar{d}\ \bar{t}=$ 



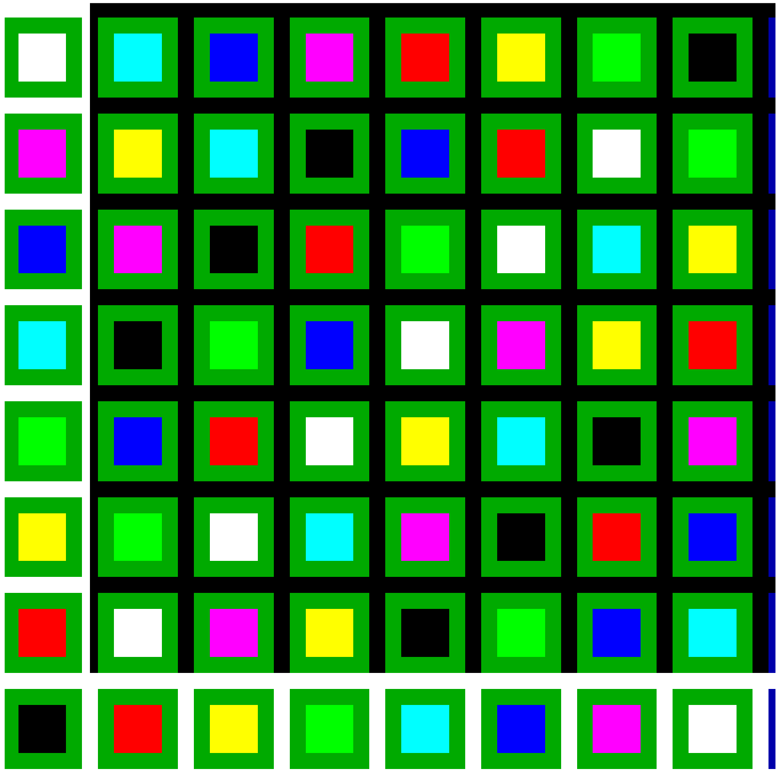
"Half-Bosons" multiplication table using a flavor/color couple:

- **Signature (by identifying the first line)**

The tritmap form of the 48 bosons reveal their multiplicative and additive actions. But 16 trits is too much too encode 48 items, only 4 trits are needed.
The tritmap as a square is naturally ready to stay in a 2D geometrical network. On the price of the information redondancy they are geometrically friendly. Id est they can be uniquely recognized only by analyzing one line, or one column. For example, the first line of each of the 48 tritmaps will give it signature, as a ternary number of four trits.

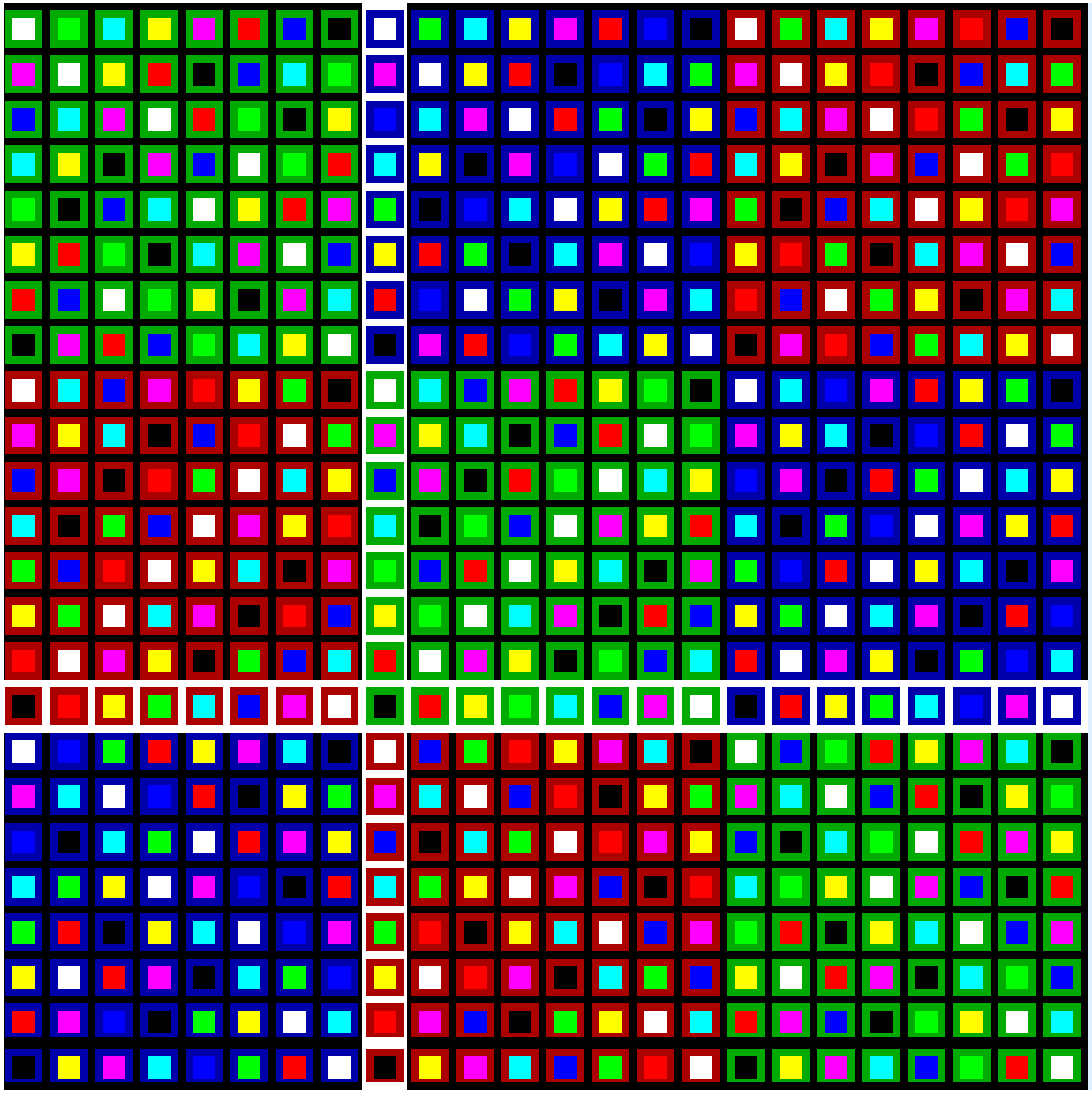
- **Color coding (based on darkgreen flavor)**

- **Zoom on: 8 higgsons (of darkgreen flavor, basis for colors) product table**



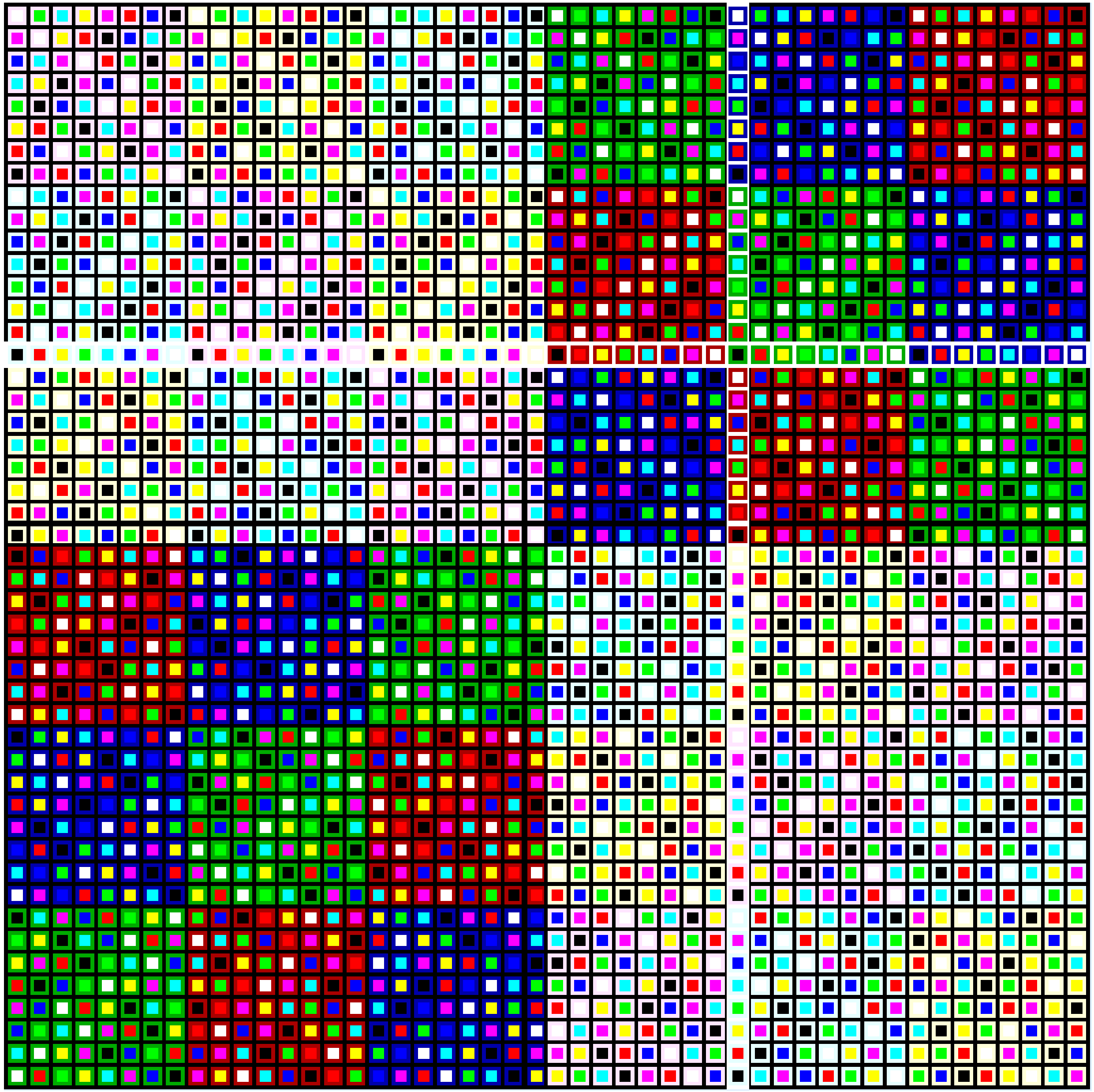
"Half-Bosons" multiplication table using a flavor/color couple :

- **Zoom on: 24 higgsons (of dark flavors) product table**



"Half-Bosons" multiplication table using a flavor/color couple :

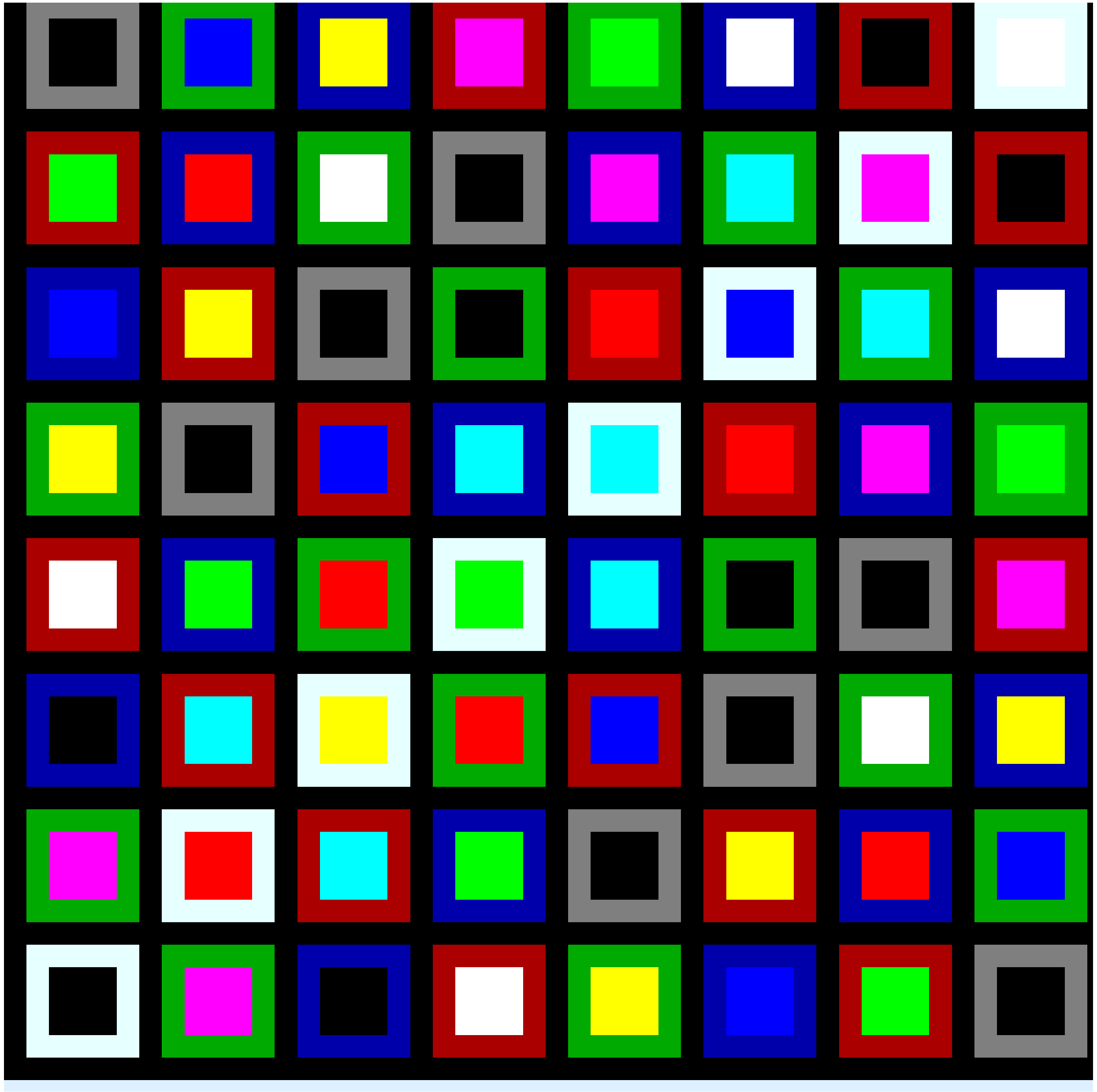
- **Complete 48 half-bosons product table**



"Half-Bosons" addition table using a flavor/color couple:

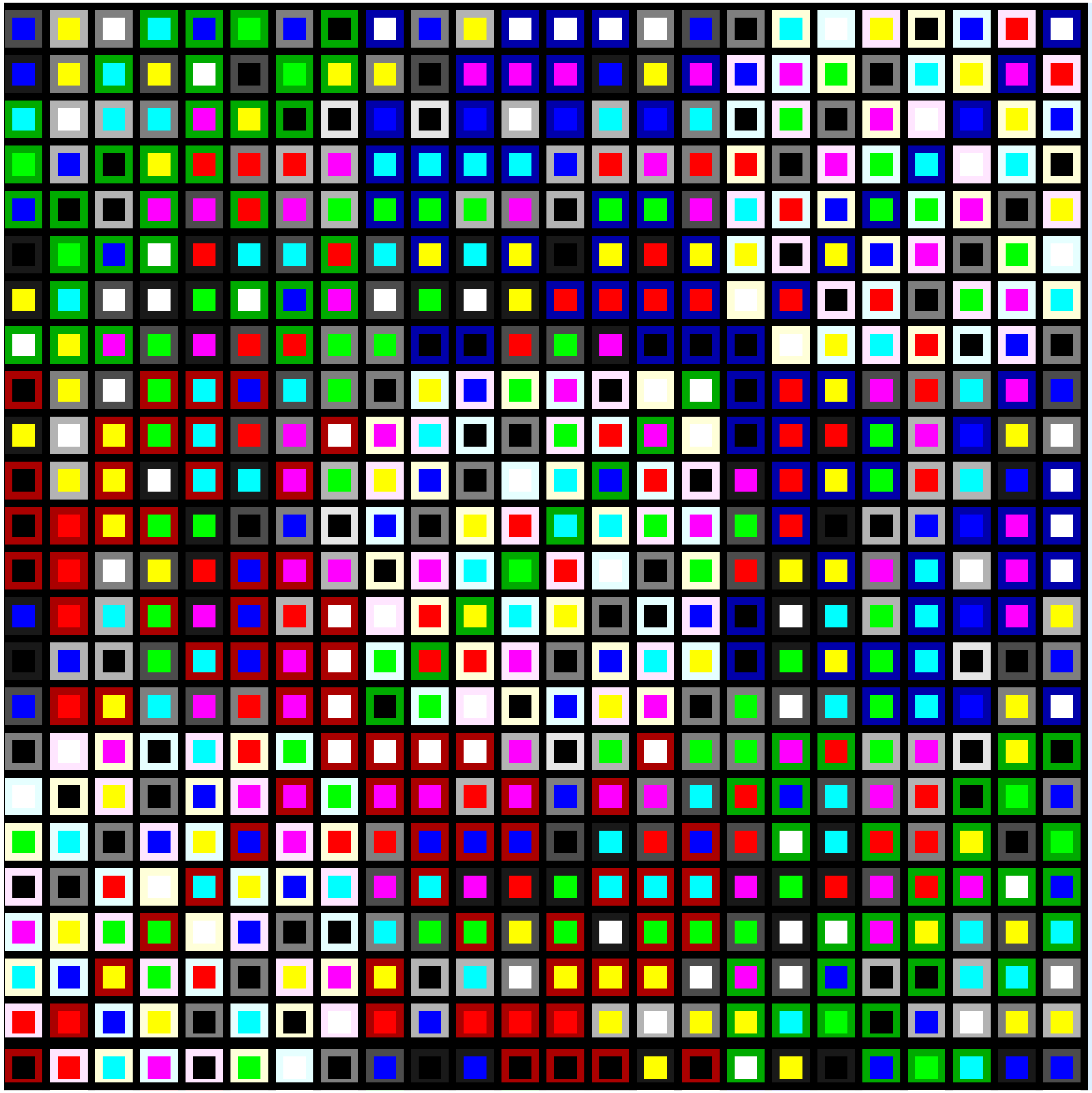
- **Zoom on: self interaction of 8 gluons (of lightcyan flavor)**

red on light cyan is blue+antigreen gluon, cyan on light cyan is green+antiblue gluon, they annihilates together and gives photons (black on gray)



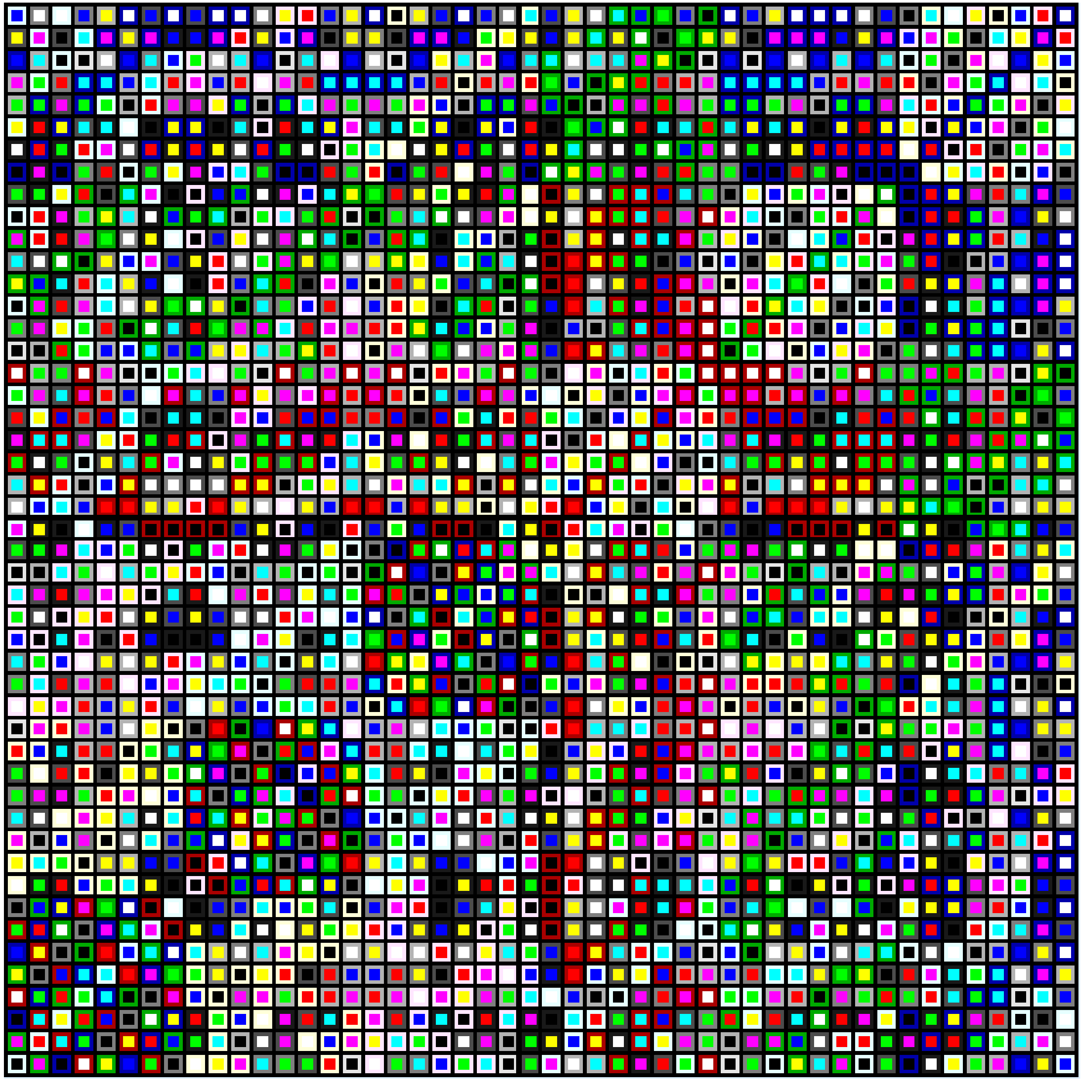
"Half-Bosons" addition table using a flavor/color couple :

- **Zoom on: 24 higgsons (of dark flavors) addition table**

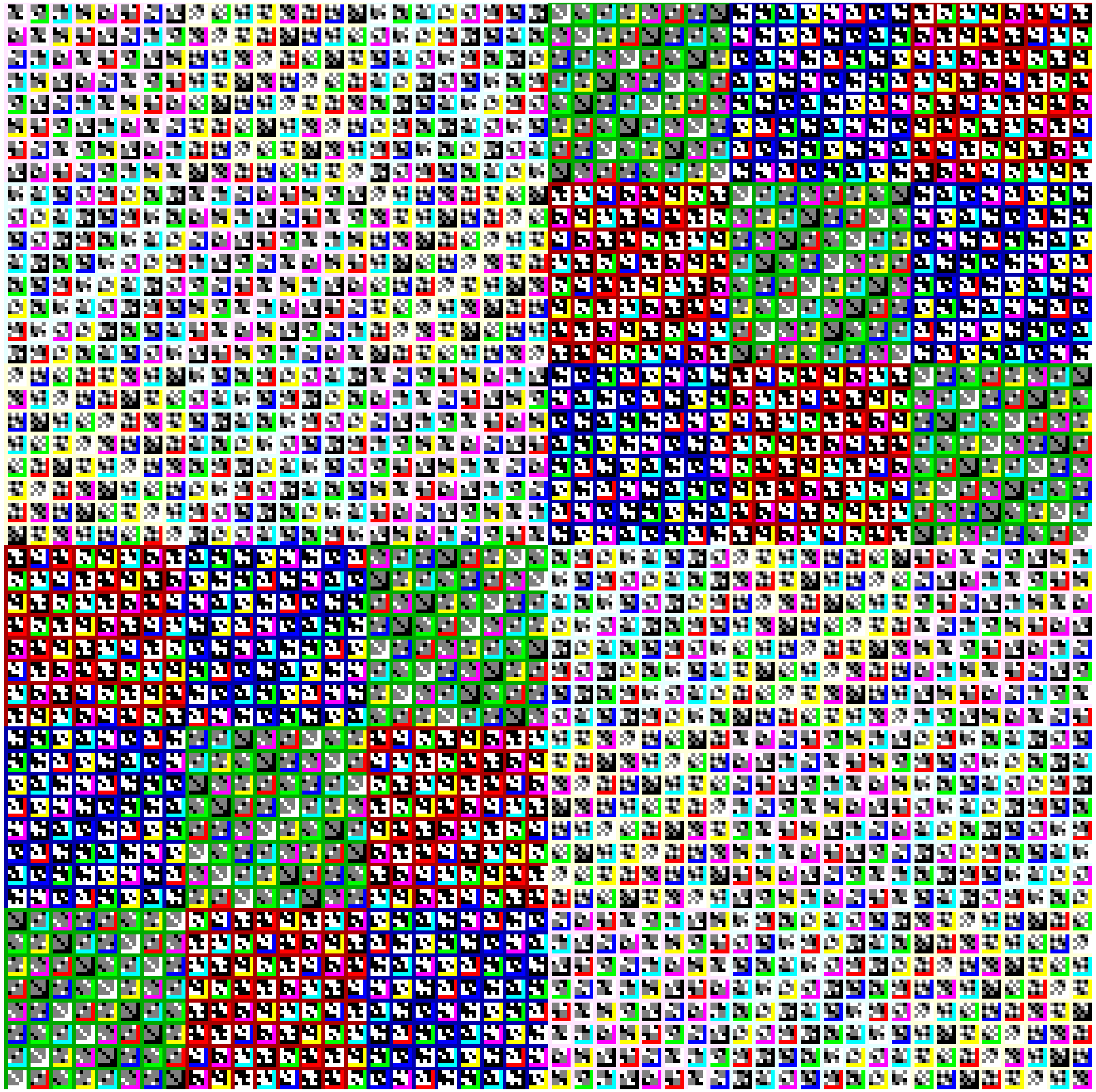


"Half-Bosons" addition table using a flavor/color couple :

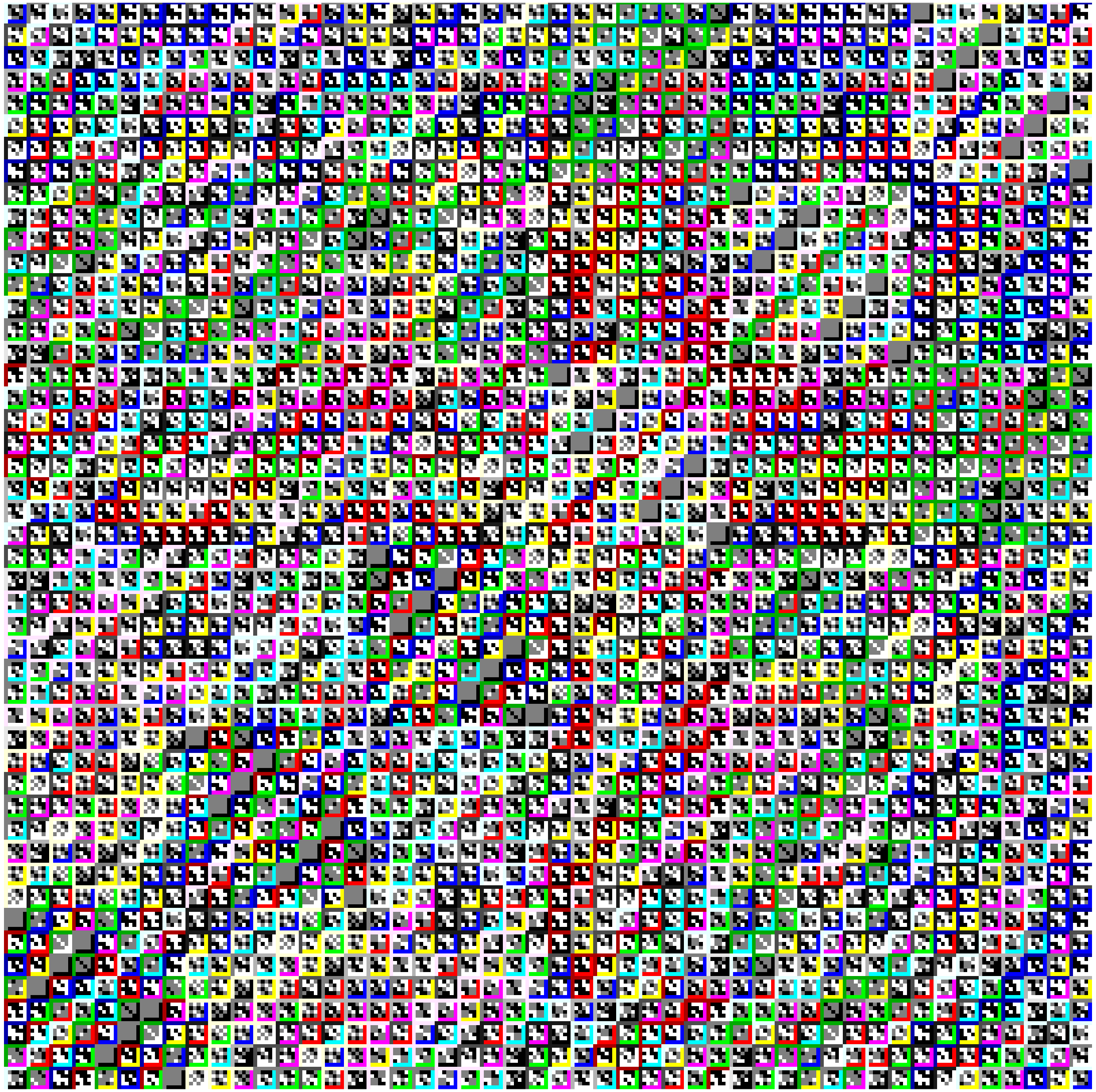
- **Complete 48 half-bosons addition table**



half-bosons multiplication table

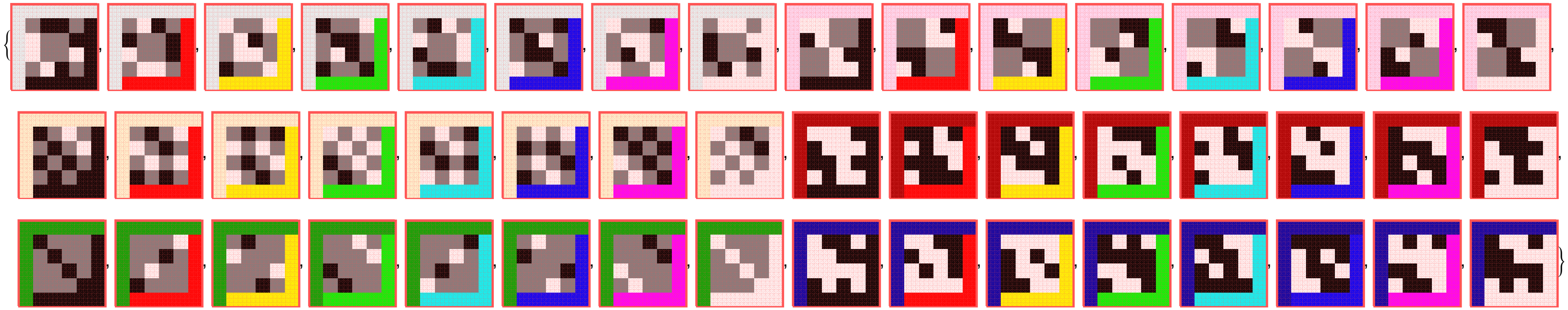


half-bosons addition table

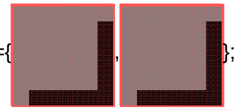
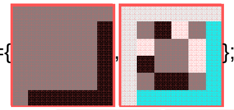
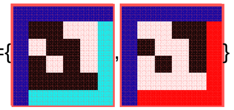


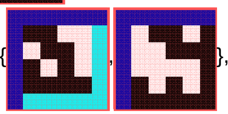
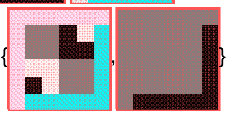
2) "In the end the machinery will be revealed"
2.1) Machinery components: e8 roots

- **tritmaps generated by yin and yang and extended by triality and duality build the multiplicative group of 48 half-bosons.**
- **The neutral element of tritmaps sum is the 49th half-boson, the half-photon.**
- **We get E8 roots by pairing half-bosons, when :**
- **Ordered pairs of {half-photon, half-boson light flavor} gives 24 bosons of gluonic sector**
- **Ordered pairs of {half-photon, half-photon} gives the unique self-antiparticle: the photon**
- **Ordered pairs of {half-boson light flavor, half-photon} gives 24 bosons of higgsonic sector**
- **Ordered pairs of {half-boson dark flavor, half-boson dark flavor} of same flavors gives 3*64 fermions**
- **First line ternary units gives roots coordinates in E8 (after scaling)**
- **Samples:**

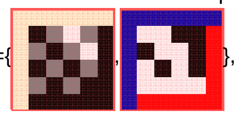
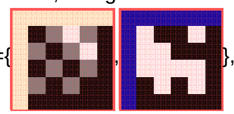
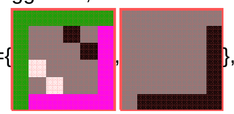
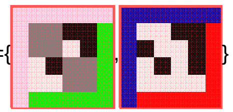
48 half-bosons = {}

First line ternary units gives roots coordinates in E8 (after scaling), using (T,S,U,V,w,x,y,z) basis, Table 9 [Lisi]

Photon={}; Green/Antiblue Gluon={};Blue up quark left spin+={}

Red up quark left spin+={}, W+ boson={}

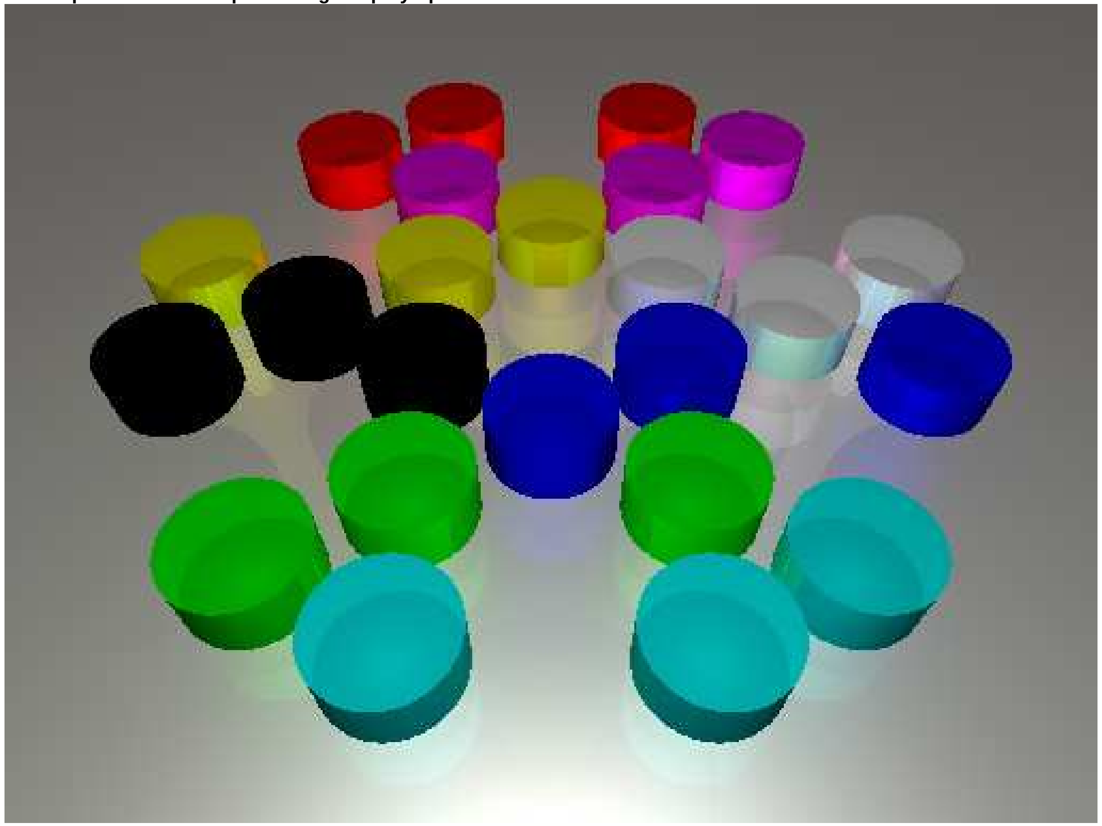
Alternatively, we can choose to work with more physical (L,R,W,B,w,x,y,z) basis, using dual half-boson for the higgsonic, in this basis:

Blue up quark left spin+={}, Red up quark left spin+={}, W+ boson={}, Blue top quark left spin+={}

Charge Q=W+B(llightyellow)+R(lightmagenta)+L(lightcyan)+1/3(x+y+z), respectively -2/3,-2/3,1,-2/3

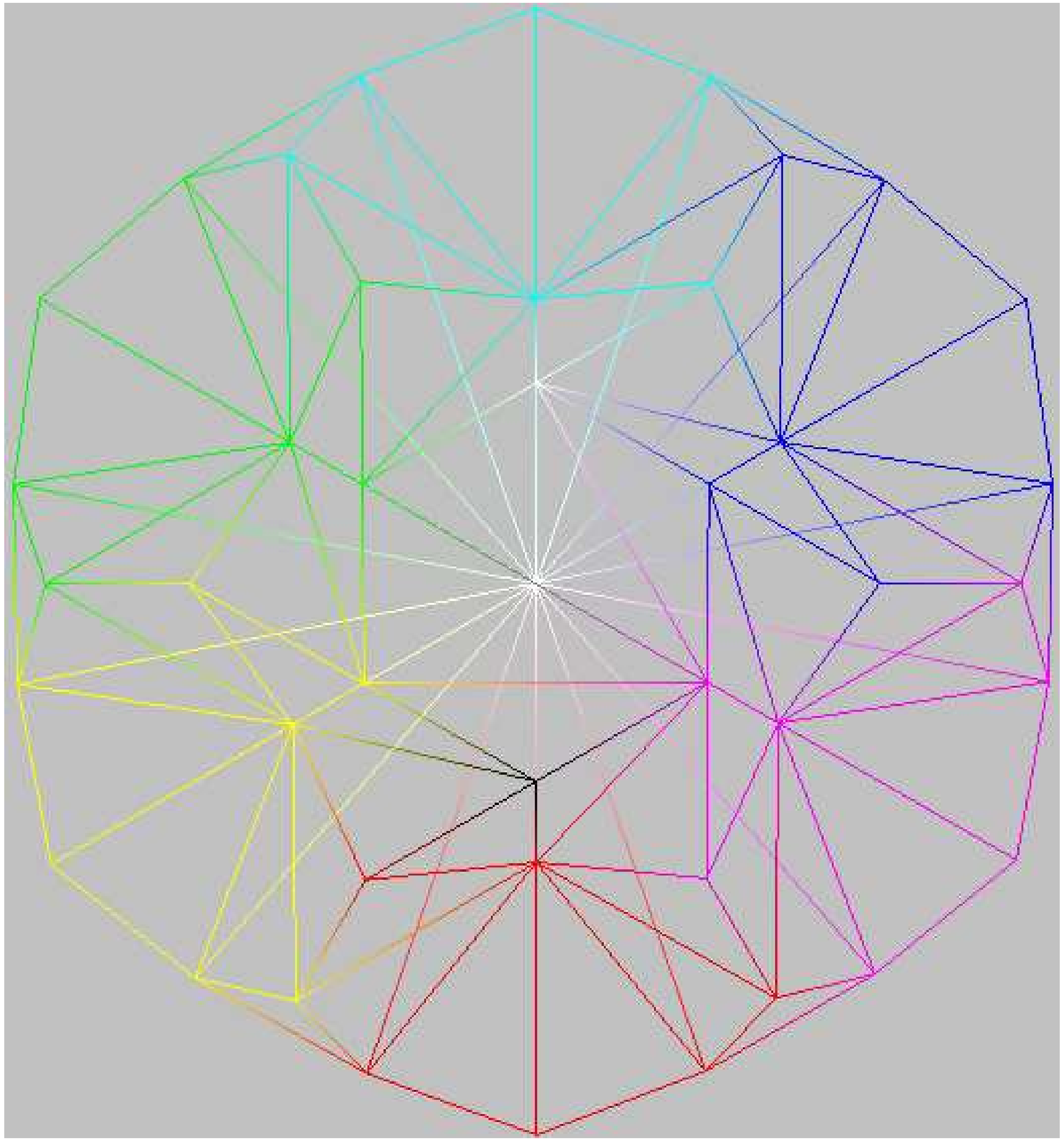
2.2) Machinery Architecture: 24-Cell

- **24-Cell is the unique self-dual exceptional regular polytope.**



Each particle is made of a couple of a 24-cell vertex and a 24-cell-dual vertex (or center for the bosons)

▪

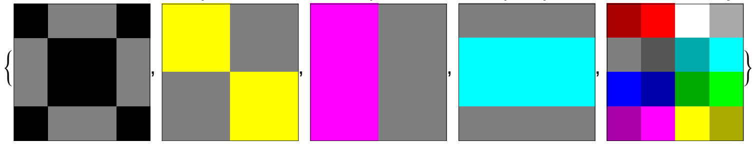


3) "The laws will turn out to be simple, like the chequer board"
3.1) Checker board

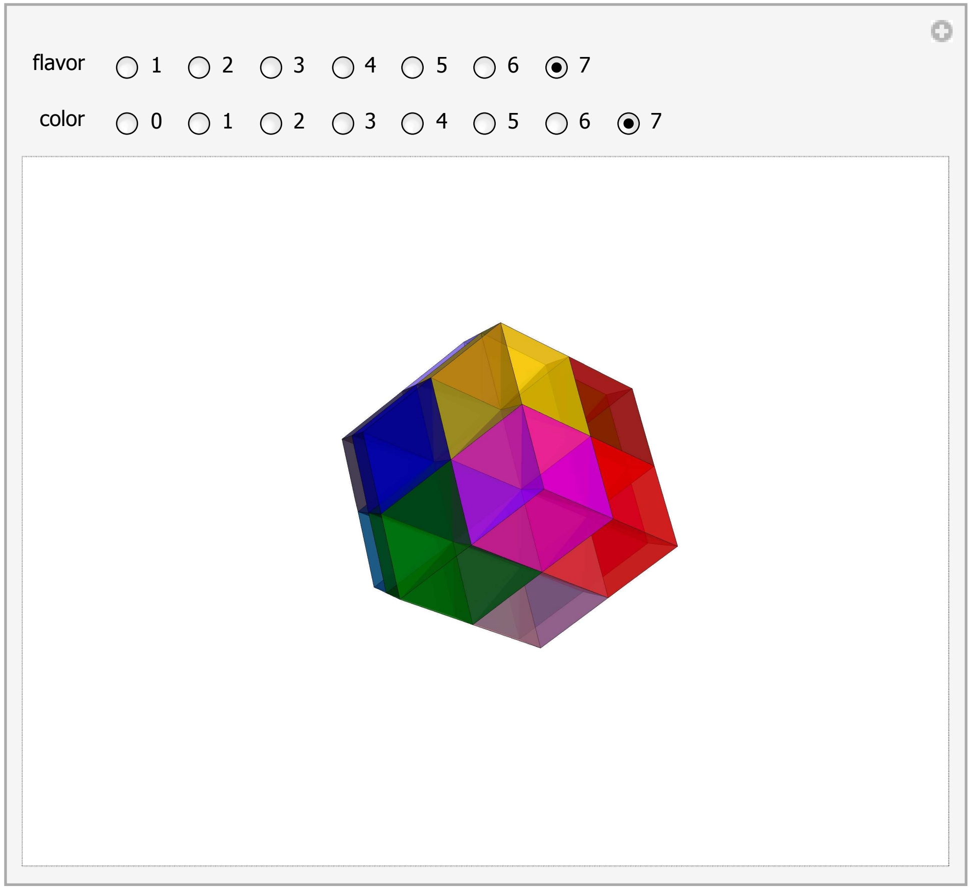
- **Duality determines checkerboard class (dark->black or light->white) of each half particle.**
- **Each particle is made of two adjacent cases in a checkerboard**
  Boson={dark,light}, Fermion={light,dark}.

3.2) Geometry: 4D Elementary Cellular Automata

We will map the sixteen dots to sixteen vertices of an hypercube.
We choose 4 decompositions of the square in two equal parts, each for every basis color of our four dimensionnal space



- **Quadridimensional cell**



Hyper checkerboard neighbourhood

in 2D: square is in contact with same color squares at its four vertices
in 3D: cube is in contact with same color cubes at its 12 edges
in 4D: hypercube is in contact with same color hypercubes at its 24 faces

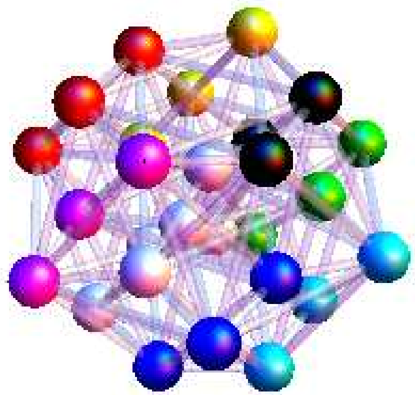
Hypercube cut by 4 perpendicular hyperplanes intersecting at center has four dots at each face.
Four dots are enough to recognize a half-boson signature.

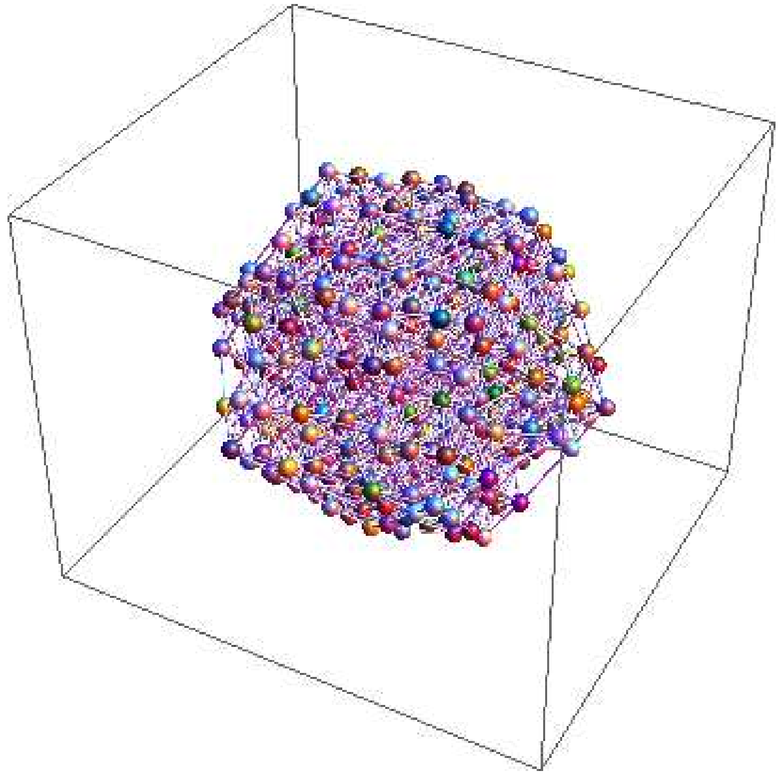
An hypercubic checkerboard lattice made of white cells for lightflavor higgsonic fermion sector and darkflavor higgsons, and black cells for darkflavor gluonic fermion sector and lightflavor gluons is a simple geometry allowing E8 internal symmetry for particles.

4D symmetry may be broken to associate a couple of white and black neighbours cells along one direction, the time direction, and identify them as particle.

NKS Network

4D lattice build from two sets, an even and an odd.





Particles location is the even set. Particle caracterization as a couple of half-bosons is made by removing one link to one of 24 neighbour vertices of the odd set, defining the gluonic half-boson, and one link to one of 24 neighbour vertices of the even set, definig the higgsonic half-boson.

In geometric space, gluonic links are of length 1 like {0,0,0,0}→{$\frac{1}{2}$,$\frac{1}{2}$,$\frac{1}{2}$,$\frac{1}{2}$}; while higgsonic links are on length $\sqrt{2}$ like {0,0,0,0}→{1,1,0,0}. Basis for gluonics and higgsonics are respectively {k,y,m,c} ({w,x,y,z} in Lisi notation), and {S,T,U,V}, so positions in the 24-cell dual like {1,1,0,0} are basis of the dual basis {L,R,ω,B}.
Triality contraint can reduce the network topology.
Without triality, the network topology is:
each odd node is connected to 16 odd neighbours and 8 even neighbours at distance 1
each even node is connected to 24 odd neighbours at distance 1 and 24 even neighbours at distance $\sqrt{2}$
Replacing each even node by a triplet (at same position) linked each to only 8 odd neighbours at distance 1 and 8 even neighbours at distance $\sqrt{2}$ (connections restricted to same triality) keep only representations of 192 observed fermions of E8.
This lattice is a 4 dimensional version of the diamond lattice. We can see it simply as a cristal made of 4D bubbles whose boundaries are the edges of the 24-cell 4D regular polytope, linking 24 vertices on an hyper-sphere S3, belonging to the odd set, and whose centers are the even set of triplets with links starring to the bubble and the neighbours centers.
Using topological projection (build by GraphPlot3D), we can project it in 3D, forgetting original 4D coordinates.
On this cristal, a fermion is repesented by the lacking of two links at one even node, one to the shell and one to the neighbourhood, while a boson is just the lacking of one link at one even node.
This show how Ockam's razor has been used to implement E8 theory in a NKS network paradigm.
This crystal may be the just simple enough, and sophisticated enough, network structure evocated in NKS.
Describing this model to Stephen Wolfram, he remarked that it would be better if this crystal could in some manner be self generated instead of being preexistent. In Nature, crystals are always results of growing, so this one, while including all Nature, should do so. Its architecture is imposed by uniqueness of some exceptional mathematical objects like the E8 Lie group and the 24-Cell polytope. This unique mathematical rule can be the external contraint fixing the structure of the growing network without using any external physics.

```
oddSet = {x = {a, b, c, d} | And[And[IntegerQ[2 a], IntegerQ[2 b], IntegerQ[2 c], IntegerQ[2 d]],
      Or[And[OddQ[2 a], OddQ[2 b], OddQ[2 c], OddQ[2 d]], And[IntegerQ[a], IntegerQ[b], IntegerQ[c], IntegerQ[d], OddQ[a + b + c + d]]]]};
evenSet = {x = {a, b, c, d} | And[IntegerQ[a], IntegerQ[b], IntegerQ[c], IntegerQ[d], EvenQ[a + b + c + d]]};
oddNetwork = {x → y | And[x ∈ oddSet, y ∈ oddSet, Norm[x - y] == 1]};
evenNetwork = {x → y | And[x ∈ evenSet, y ∈ evenSet, Norm[x - y] == √2]};
coupledNetwork = {x → y | And[x ∈ evenSet, y ∈ oddSet, Norm[x - y] == 1]};
```

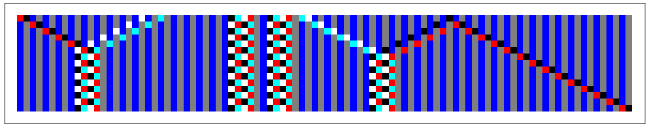
3.3) Rules:

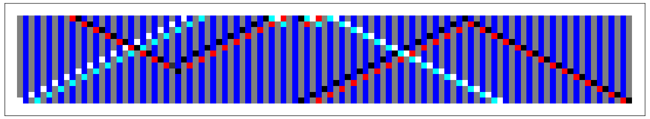
- **3D Elementary Cellular Automata of an Hypercube projection along time axis**

Classical image of a cube in a cube.

Internal cube may contain eight dots coding past state, while external cube codes present state.

3D second-order ECA.





Phased time, (cf Ed Fredkin). Six colored phases following rainbow cycle

$= \{-\mathbb{k} \quad \mathbb{i} \quad -\mathbb{j} \quad \mathbb{k} - \mathbb{i} \quad \mathbb{j}\} = \{$  $\}$

Each phase fix a direction and operate in the perpendicular 2 D plane. Colored and anti – colored phases are alternating, each acting on respectively black or white sectors (gluonic or higgsonic).

Possible adaptation of Miller – Fredkin RCA, yet with three states.

- **Digital General Relativity applied to the Crystal network**

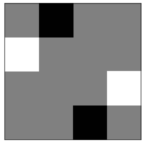
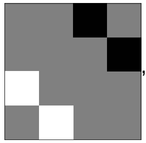
In NKS 9th chapter, some approach to compute Ricci curvature for a network are given, and seems natural when this network has, like our cristal, dimension four, even it is more complex than in dimension two.
Other works from Forman and Sullivan gives also means of computing curvatures in discrete manifolds.
Discrete curvature is not uniquely defined actually, so we have to do virtual experiments with our model.

This 4D network can be turned in a causal network 4D spacetime by orienting the links. The local structure of 24-cell and its dual to generate E8 symmetry and the standard model remains, as it is now a topological feature.
The real 4D spacetime coordinates are no more the initial coordinates used to build the cristal. They appear as a result of cutting spacelike slices in an event cone an obey special and general relativity as explained in NKS. New frame-higgs bosons introduced by Lisi gives local curvature to the crystallic topological network, and from this curvature, a digital Lagrangian can be build and Einstein equation can holds so a new TOE is made available in the NKS paradigm of an easy to understand concept.

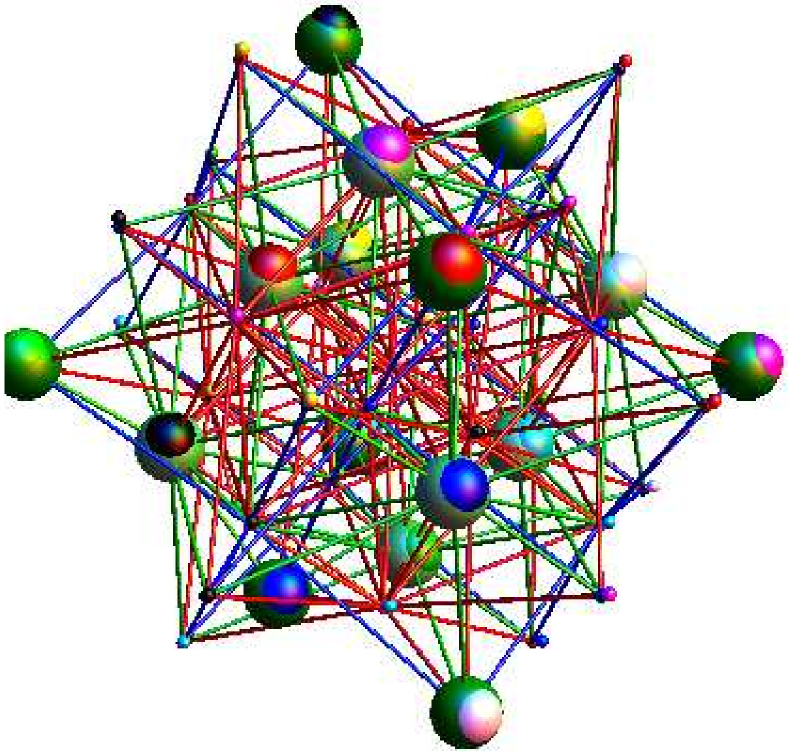
- **Crystal network as Non Associative Space**

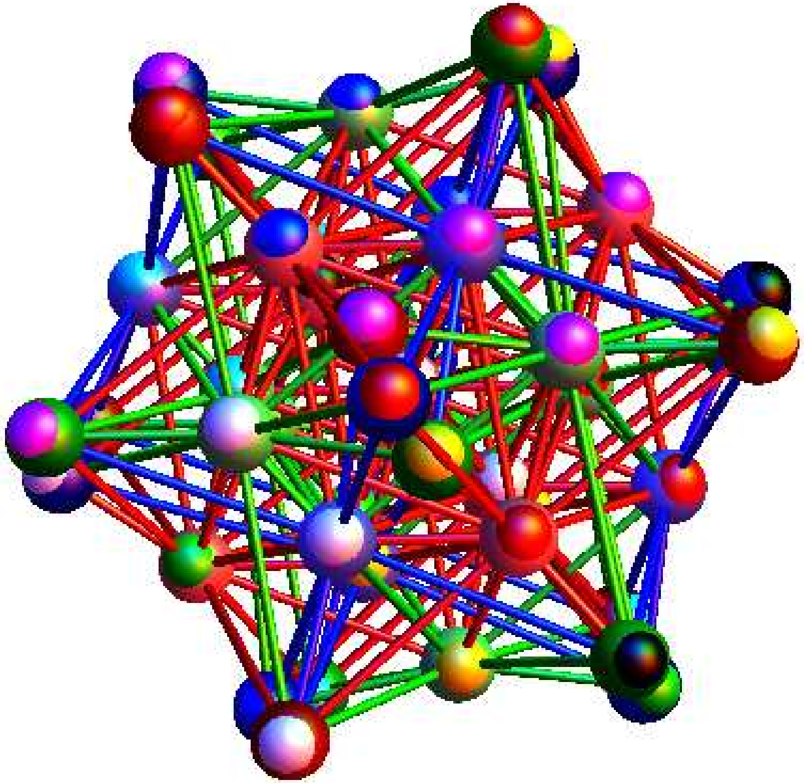
Lisi's E8 theory, Wolfram's Causal network, and Connes's Non Commutative Geometry are three complementary visions toward a Toe. Presented work is related to Non Associative Geometry, à la Wulkenhaar, by replacing Connes H+H+M3(C) finite non commutative space by 24-cell + its dual non associative one, linking directly to standard model through e8, and effectively a causal network.

Conclusion

1) From 2 generators,  and , without mathematical statements, we went to 240 particles of Lisi's E8 Theory of Everything (which includes standard model and gravitation).

2) A tentative to replace the BF action from MacDowell and Mansouri which links Lisi's model to more standard physics by a Digital Mechanics linking it to NKS bring us to a checkerboard lattice model, as a four dimensional crystal which is a topological causal network embedding this unique four dimensional self dual exceptional polytope, the 24-cell.
In this NKS paradigm, the universe is just a topological hyperdiamond.
This gives a new rational to why we all like so much diamonds...





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