1. The circumference of a circle is $2\pi$. The perimeter of a hexagon inside the circle will be less than $2\pi$ and the perimeter of a hexagon outside the circle will be greater than $2\pi$. What are the best bounds on $\pi$ that can be obtained from simple use of these ideas?

2. Simplify $\sum_{1 \leq i \leq n} \sum_{0 \leq j \leq i} \frac{j}{i}$.

3. The recurrence $E_n = E_{n-1}^2/2^n$ with the boundary condition $E_0 = 1$ is nonlinear. However, if you take the logarithm of the equation, you obtain a linear first order equation in $\log E_n$.
   a. Show the equation that results from taking the logarithm. (You may choose any useful base for your logarithm.)
   b. Solve for $E_n$.

4. Let $a_{n+1} = (a_n + b_n)/2$ and $b_{n+1} = \sqrt{a_nb_n}$. Define $\epsilon_n = b_n - a_n$.
   a. Express $b_n$, $a_{n+1}$, and $b_{n+1}$ exactly in terms of $a_n$ and $\epsilon_n$.

   b. Express $b_n$, $a_{n+1}$, and $b_{n+1}$ exactly in terms of $a_n$ and with a power series of cubic order in terms of $\epsilon_n$, that is the least significant term should have the form $f(a_n)O(\epsilon^3)$ for $\epsilon_n$ near 0, where $f$ is some function of one variable.

5. Assume $x^2e^x = t$. Find an asymptotic solution for $x$ as a function of $t$ that is correct for large $t$. 