1. The random integer $a$ is added to the fixed integer $j$. In each case below, what is the probability that $a + j \geq 2^k$?
   a. $a$ is equally likely to be any integer in the range $0 \leq a < 2^k$.
   b. $a$ is equally likely to be any integer in the range $2^{k-1} \leq a < 2^k$.

2. The random integer $a$ is added to the random integer $b$. For each case below, what is the probability that the sum requires at least $k+1$ bits to write in base 2, when the sum written with no leading zeros?
   a. $a$ is equally likely to be any integer that can be written with $k$ bits, possibly with leading zeros; independently, $b$ is equally likely to be any integer that can be written with $k$ bits, possibly with leading zeros.
   b. $a$ is equally likely to be any integer that can be written with $k$ bits, possibly with leading zeros; independently, $b$ is equally likely to be any integer that can be written with $k$ bits, without leading zeros.
   c. $a$ is equally likely to be any integer that can be written with $k$ bits, without leading zeros; independently, $b$ is equally likely to be any integer that can be written with $k$ bits, possibly with leading zeros.
   d. $a$ is equally likely to be any integer that can be written with $k$ bits, without leading zeros; independently, $b$ is equally likely to be any integer that can be written with $k$ bits, without leading zeros.

3. The Generalize Euler Summation Formula (eq. 4-164–165) gives a way to exactly compute
\[
\sum_{0 \leq i \leq n} i^k
\]
as a sum of approximately $k$ terms involving Bernoulli numbers. Give a derivation of the formula.

4. Consider the recurrence
\[
T_n = 5T_{n-1} - 8T_{n-2} + 4T_{n-3}.
\]
   a. Give a general solution to the recurrence.
   b. Give the particular solution that obeys the boundary conditions
      \[
      T_0 = 1, \quad T_1 = 2, \quad T_2 = 4.
      \]

5. Consider the equation
\[
(B^a U_2 + U_1)(B^b V_2 + V_1) = B^c U_2 V_2 + B^f (U_1 + B^c U_2)(V_1 + B^d V_2) - B^g U_1 V_1 - B^h U_2 V_2 + B^i U_1 V_1,
\]
where we want to find values for all the lower case letter that will make the equation hold for all values of the upper case letters.
   a. Algorithm 5.2 in the text gives a version of Karatsuba’s algorithm which is based on a particular solution to these equations. In terms of $n$, which value for $a, b, c, d, e, f, g, h, i$ does Algorithm 5.2 use?
   b. Give the most general solution (or set of solutions) to these equations that you can find. To avoid some useless trivial solutions, you may assume that $a > 0$ and $b > 0$ if you so state.