Errata to The Analysis of Algorithms, First Printing.

Those corrections to the first printing that were made in the second printing are given.

Usually just the corrected segment of text is given. Negative line numbers indicate the number of lines from the bottom.


p 7, Fig. There should be an arrow from node 3 to node 2, and there should be no arrow from node 4 to node 2.

p 15, ex. 3. does not imply

p 21, l 13. that are set in the next


p 39, l 8. \( n \), we have that \( g(n) = O(f(n)) \)

p 47, l 12. placed on the statements.

p 51, l 11 to 16. (for which \( n \geq 2m \)) to smaller cases: if \( n \) is even, then \( C_{mn} \) is obtained from \( C_{m,n/2} \), while if \( n \) is odd, it is obtained from \( C_{m,(n-1)/2} \). If \( n \geq 2m \) [so that eq. (95) or eq. (96) must be applied], then \( n/2 \geq m \) [and \( (n-1)/2 \geq m \) if \( n \) is odd]. We can therefore use the partial well-order defined by \( (m, n_2) < (m, n_1) \) if and only if \( n_2 < n_1 \).

When \( m \leq n < 2m \), the theorem is proved directly. When \( n \geq 2m \),

p 80, l 1. where \( j < m \).

p 92. new exercise.

p 98, l 2. up to \( 2p \).

p 112, ex. 2. upper limit

p 113, l 6. replaced

p 115, l 12. up to \( k \).

p 115, l 4. each pair \((i, j)\)

p 121, l 7. referenced

p 121, l 10. referenced

p 125, ex. 2. unity and \( 0 \leq m < k \), then

p 129, eq. (167). integer \( n, r \), \( r \geq 0 \).

p 134, l 4. integers taken from 1 to \( n - i \); that is,

\[
\left\{ \frac{n}{n - k} \right\} = \sum_{1 \leq i_1 \leq i_2 \leq \cdots \leq i_k \leq n-k} i_1 i_2 \cdots i_k. \tag{186}
\]

p 135, eq. (193).

\[
\sum_{i} \left( \frac{n}{i} \right) \left( \frac{i}{m} \right) = \left\{ \frac{n + 1}{m + 1} \right\}. \tag{193}
\]

p 140, Table. \( 2F_1 \left[ \frac{1,1;z}{z} \right] \ln(1-z) \), \( 2F_1 \left[ \frac{1,1;-z^2}{z} \right] \arctan z \).

p 154, l 5. considerations lead to the idea

p 155, ex. 5. Prove or disprove that the binomial tree algorithm is optimal for destructive testing.
p 155, following eq. (25). The function \( f(x) \) in these definitions is assumed to be nonnegative.

p 157, l 1. (provided \( f(x) \) does not change sign in the region of interest). In other words,

p 158, ex. 9. Show that if \( g(x) \geq 0 \) and \( f(x) = \Omega(g(x)) \),

p 159, l 19. if a power series

\[
\left[ 1 - \frac{t^2}{2n} + O\left(\frac{t^3}{n^{3/2}}\right) \right]^n = \exp \left[ -\frac{t^2}{2} + O\left(\frac{t^3}{n^{1/2}}\right) \right].
\]

p 160, ex. 7.

\[
\sum_{1 \leq i \leq N} |\lg i| = (N + 1)|\lg N| - 2|\lg N| + 1 + 2.
\]

\[
N \lg N - 3N + \lg N + 1 < \sum_{1 \leq i \leq N} \lg i < N \lg N + \lg N + 2
\]

\[
16(N/8)^{N+1} < N! < 4N^{N+1}
\]

p 168, ex. 2. Delete this exercise.

p 180, ex. 4. Simplify eq. (111)

p 180, ex. 5. Simplify eq. (111)

p 187, l 4. For \( m \geq 1 \), we can use integration by parts on eq. (150) and sum the results to obtain

p 188, ex. 2. Drop the \( n^{-5} \) terms in the upper limits and change the constants to 1.6667, 1.6458, and 1.6451 (for \( k = 1, 2, 3 \)).

p 188

3. Simplify \( \sum_{0 \leq i \leq n} i^k \) for \( k \leq n \).

p 190, l 10. but for large even \( i \), \( |B_i| \)

p 194, eq. (198).

\[
\lim_{n \to \infty} n \left( \frac{(2n)!}{4^n n!} \right)^2 = \frac{\Gamma(1)\Gamma(1)}{2\Gamma(1/2)\Gamma(3/2)} = \frac{1}{\pi}
\]

(198)

p 195, eq. (203). where \( a_i > a_{i+1} > 0 \),

p 203, l 1. whereas

p 209, eq. (70) \( 4 \cdot 3^k \)

p 210, ex. 4

\[
T_{2n+1} = t_1 + \frac{2n}{2n + 1}T_n
\]

with boundary condition \( T_1 = t_2 \), where \( t_1 \) and \( t_2 \) are constants.

p 213, l 16. \((n+1)\)-bit

p 216, eq. (102).

\[
T_n = \frac{a(n+1)}{2} \lg \frac{n + 1}{2} + \frac{b - a + c}{2}n + \frac{c - a - b}{2}.
\]

(102)

p 219, eq. (104).

\[
a_n = \frac{1}{n!} \left. \frac{d^n G(z)}{dz^n} \right|_{z=0}.
\]

(104)
p 228, Algorithm 5.7. A list $A$ of input tapes.

p 228, Algorithm 5.7. Step 4. Write $a_n$ onto Tape $B$.

p 230, 11. On the last phase we want to be merging two tapes each of which contains

p 232, 1–4. Applying eq. (157) gives

p 234, eq. (179) = $b_n$.

p 235, 1–7. $b_n$.

p 235, 1–5. $b_n$.

p 240, ex. 4. $T_n = 4T_{n-1} + 5T_{n-2}$.

p 245, 1–17. group. (This takes 19 comparisons.

p 245, 1–15. contain

p 246, Step 5. median element

p 246, Step 5. We will call the element of the small group with

p 248, eq. (252).

\[
C(n_0) \leq \frac{1}{9} \alpha n_0 + \frac{1}{3} \beta + \alpha + \beta + \frac{13}{18} \alpha n_0 + \frac{13}{18} \beta + 4 \alpha + 4 \beta + \frac{22}{9} n_0.
\]  

(525)

p 248, 13. $1 + \left[ \left\lfloor \log((n \mod c)/2) \right\rfloor \right]$ comparisons.

p 248, eq. (248)

\[
(1 + \left\lfloor \log((c - 1)/2) \right\rfloor) \left( \left\lfloor \frac{n}{c} \right\rfloor - 1 \right) + 1 + \left\lfloor\log((n \mod c)/2)\right\rfloor \leq \frac{1 + \left\lfloor \log((c - 1)/2) \right\rfloor}{c} n
\]  

(248)

p 248, eq. (253).

\[
\alpha n_0 + \beta \geq \frac{1}{3} \alpha n_0 + \frac{1}{3} \beta + \alpha + \beta + \frac{13}{18} \alpha n_0 + \frac{13}{18} \beta + 4 \alpha + 4 \beta + \frac{22}{9} n_0.
\]  

(253)

p 250, ex. 1. 13 comparisons; to sort 11 elements requires 26 comparisons.)

p 252, eq. (8). $\sum_{k \geq 1} p_k z^k = \sum_{k \geq 0} p_k z^k - p_0 = G(z) - p_0$.

p 254, eq. (23).

\[
p_k(t+dt) = (1 - \lambda dt - k\mu dt)p_k(t) + [(k+1)\mu dt]p_{k+1}(t) + (\lambda dt)p_{k-1}(t) + o(dt)
\]  

(23)

p 256, ex. 4. Show that the $F(n) = 2F_1(n, b + a/(c + 1); b; c + 1)$ is a solution to the recurrence $n c F_{n+1} + [(1 - c)n + a] F_n + (b - n) F_{n-1} = 0$.

p 258, 11. Thus, when $a_k(i) \neq 0$ for $k \leq i \leq n - k - 1$.

p 260, eq. (55).

\[
\sum_{1 \leq i \leq n} D(i)
\]

(25)

p 261, ex. 3. Solve $F_{n+2} - \frac{2n + 1}{n} F_{n+1} + \frac{n}{n-1} F_n = n(n + 1)$. Hint: $F_n = n - 1$ is a solution of the homogeneous equation.

p 261, eq. (57) and (58). $\sum_{0 \leq i \leq k-1} \sum_{0 \leq i \leq k}$.

p 261, 1–8. Now, $\sum_{0 \leq i \leq k} a_i(n)E^i$.

p 266, 1–9, –7, –5. Change eq. (100) to eq. (99).

p 267, eq. (109). The right vertical line for the determinant is missing.

p 283, ex. 1. Delete this exercise.

p 291, ex. 1. Delete this exercise.
p 292, ex. 3. \( L(x) = \frac{C(x) - xC(x) - 1}{1 - 2xC(x)} \).

p 296, eq. (95). Replace \((t-1)!\) with \(t!\).

p 296, l 8. A factor of about \((1-p)^p\)

p 296, ex. 2–4. Move these exercises to section 7.2 (p 301).

p 296, ex. 4. \( T_n = aS_n + b \) is made with the right choices for \(a\) and \(b\).

p 300, eq. (119). Bold face equation number.

p 301, l 5. Nonlinear recurrence equations that can be transformed into linear recurrences are sometimes called pseudo-nonlinear recurrences. They are considered in

p 303, l 3.

**EXERCISE**

1. Improve the error bound on eq. (138) by improving the approximation leading to eq. (135). Hint: Use \( \ln x \leq (x/n) \ln n \) for \( x \geq n \).

p 305, table. \( K^{2^n} \).

p 306, ex. 2. \( X_n = T_n + T_{n-1} \).

p 309, eq. (174). Change \( b \) to \( f \)

p 312, –13. Put in a little vertical space before the start of the paragraph.

p 316, l 1–7. By considering the properties of path length, we can now find the solution of our recurrence. Consider an old tree and a new tree where the new tree is obtained from the old tree by moving a pair of leaves (with a common parent) so that they are the children of some node that use to be a leaf node. Repeated use of this transformation can convert any \( n \) node tree where every node has zero or two children into any other such tree. The transformation creates one new leaf (the common parent in the old tree) and converts one leaf into an internal node (the common parent in the new tree).

p 316, eq. (192).

\[
C_n = 1 + \frac{kn + 2j}{n} = \lfloor \ln n \rfloor + 3 - \frac{2^{\lfloor \ln n \rfloor + 1}}{n}.
\]

p 320, ex. 1. \( \frac{1}{3} T_{i+1} + \frac{1}{3} T_{i-1} \) when \( i \) is odd

p 327, l 9. Reference 81 will become 80.

p 328, l 13. \( C_{i,j} = p_i + p_{i+1} + \cdots + p_j + \min_{i<k \leq j} \{ C_{i,k-1} + C_{k,j} \} \),


\[
t = \frac{\ln M_t}{\ln \left( \frac{3 + \sqrt{5}}{2} \right)} - \frac{\ln \left( \frac{10}{3 + \sqrt{5}} \right)}{2} + O \left( M_t^{\ln(3-\sqrt{5})/(3+\sqrt{5})/\ln(3+\sqrt{5)/2}) \right) \quad (35)
\]

\[
t = \frac{\ln M_t}{\ln \left( \frac{3 + \sqrt{5}}{2} \right)} - \frac{\ln \left( \frac{5 + \sqrt{5}}{3 + \sqrt{5}} \right)}{2} + O \left( M_t^{\ln(1.9999)} \right) \approx 1.039 \ln M_t - 0.335. \quad (36)
\]

p 340, –4. The number of blocks produced
\[ P_1(t + 1) = p_{11}P_1(t) + p_{21}P_2(t) + \cdots + p_{n1}P_n(t), \quad (107) \]
\[ P_2(t + 1) = p_{12}P_1(t) + p_{22}P_2(t) + \cdots + p_{n2}P_n(t), \quad (108) \]
\[ \vdots \]
\[ P_n(t + 1) = p_{1n}P_1(t) + p_{2n}P_2(t) + \cdots + p_{nn}P_n(t), \quad (109) \]

\[ P_1(t) = 1 \]
\[ P_1(t) = 1 \]
\[ P_1(t) = 1 \]
\[ P_1(t) = 1 \]
\[ \sum_{1 \leq j \leq n} p_{ij} = 1. \]

Next we assume that there are no chains where every probability on the chain is 1. Finally, we assume that the greatest common divisor of the length of the cycles (with positive probability) of the system is one. These last two conditions are needed to avoid systems that have periodic rather than steady state behavior. (It is also possible to analyze periodic systems, but we will not do it here.)

\[ \frac{d P_0(t)}{dt} = -\mu P_0(t) + \lambda P_1(t), \quad (113) \]
\[ \frac{d P_k(t)}{dt} = \mu P_{k-1}(t) - (k\lambda - \mu)P_k(t) + (k + 1)\lambda P_{k+1}(t), \quad (114) \]
\[ \frac{d P_n(t)}{dt} = \mu P_{n-1}(t) - n\lambda P_n(t). \quad (115) \]

\[ G(x, y) = \sum_{i \geq 0, n \geq 0} p_{ni}x^ny^i. \]

Likewise, the difference of the first index of the term on the left side and
the first index of the second term on the right side should be 1:

\[ F_{n-1,i} = F_{ni}. \]

Then \( F_{n-1,i} = (n-2)F_{n-2,i} + G_{n-2,i-1} \), so \( F_{n-1,i} = (n-1)F_{n-1,i} + G_{n-1,i-1} \), which is eq. (171).]

we get that the probability that the last node added to the search tree
with \( n \) nodes has height \( i \) is

\[ (\binom{n}{k})_q = \frac{q^n - 1}{q - 1} \cdot \frac{q^{n-1} - 1}{q^2 - 1} \cdots \frac{q^{n-k+1} - 1}{q^k - 1}. \]
where \( \binom{n}{0}_q = 1 \). Prove that the Gaussian binomial coefficients are polynomials in \( q \).

p 356, ex. 5 and 6. Gaussian

p 356, ex. 7. \((1 + x)(1 + qx) \cdots (1 + q^{n-1}x)\)

p 356, ex. 9. [Rényi, see [23]]

p 356, ex. 9. Hint: First use separation of variables to remove factors of \( i \) and \( n \).

Then use the change of variable \( i' = n - i \) and the appropriate change of index

so that the equation has the standard form for three-term recurrences.

p 363, l 13. A cycle is a path where the first vertex on the path is also the last vertex

on the path. When writing the vertices of a cycle, the last vertex is omitted since

it is known to be the same as the first vertex. Each edge of a cycle connect a

vertex to the following one, and the last vertex is connected to the first one. For

example,

p 368, l 14. transformation is to make equivalent those nodes that have a common

immediate predecessor.

p 368, l 8. have a common parent, is not an equivalence relation.

p 368, l 6. a common parent.

p 369, Fig. Label the edge going into node 1 with a 0.

p 369, l 6. equivalent to each other, i.e., merge their equivalence classes.

p 370, Fig. Label the edge going into node 1 with a 0.

p 372, Fig. 9.10, 9.11. Label the edge going into node 1 with a 0.

p 376, Step 5. If \( j \leq n \)

p 376, l 13. time is \( \Theta(nm) \) (for \( m \) much less than \( n \)).

p 377, Step 6. If \( j \leq n \)

p 381.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>A</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Order(A)</td>
<td>Name</td>
</tr>
<tr>
<td>List</td>
<td>1</td>
<td>Jim</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Sue</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Ben</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Ann</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Tom</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 9.5. Possible lists for algorithms \( A \) and \( M \) along with the number of inversions of one list with respect to the other. The order(A) column gives the position of the item on the list for algorithm \( A \). The inversions column gives, for each item on list \( A \), the number of items which follow it on the list \( A \) but precede it on the list \( M \). The total of this column is the number of inversions.

p 381, l 12. Define the inversion function (for \( j > i \)) to be

p 383.
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>A</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order(A)</td>
<td>Name</td>
<td>Order(A)</td>
</tr>
<tr>
<td>List</td>
<td>1</td>
<td>Jim</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Sue</td>
</tr>
<tr>
<td></td>
<td>2 \frac{1}{2}</td>
<td>Ann</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Ben</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Tom</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

p 384, l–1. This section is based on a paper by Sleator and Tarjan
p 385, ex. 5. takes time \(O(\log_b n)\) to add 1 to the number \(n = b^k - 1\). Show that the
algorithm can be modified so that the time to start from zero and get to any
positive number \(n\) by successively adding ones is \(O(n)\).

p 389, Fig. 9.13. The trees have too many nodes and edges. Delete one edge and
one leave node from the figure for one union, delete one of the edges (and the
corresponding leaf node) on the right part of the figure for three unions, and
delete two edges (and the corresponding leaf nodes) from the figure for seven
unions (one of the leaf nodes is on the right side and three edges from the root,
and the other one is on the left side and two edges from the root).

p 389, eq. (12).

\[ H_n = 1 + \max_{0 \leq i \leq \lfloor (n-1)/2 \rfloor} \{ H_i \} \] (12)

p 389, eq. (13).

\[ H_n = 1 + H_{\lfloor (n-1)/2 \rfloor} \] (13)

p 390, eq. (16).

\[
\max_{0 \leq i \leq n} \left( \lfloor \log (i+1) \rfloor + 1 \right) (n-i) + 1 \geq \left( \lfloor \log \left( \frac{n}{2} + 1 \right) \rfloor + 1 \right) \left( \frac{n}{2} \right) + \frac{n}{2} = \Omega(n \ln n).
\]

p 391, l–13. change as Finds are done and ranks increase as you go up the tree.

p 391, eq. (23).

\[ 2m + mz + \sum_{0 \leq i \leq z} \frac{2n}{2^A_i} A_{i+1} \] (23)

p 392, l8. \( A_{i+1}/2^{A_i} \) is 1, and the total cost (for the case \( m = n \)) is bounded by
p 392, l–15. smallest \( z \geq 1 \) such that \( \log n < A(z, 4\lceil m/n \rceil) \),

p 394, l16. where \( a_j \) and \( b_j \) are interpreted as zero for \( j \geq n \) and for \( j < 0 \).
[Alternatively,

p 394, l18. and avoid any special interpretation.] The convolution and the cyclic
convolution are the same when \( B \) is a vector with period \( n \); i.e., \( b_k = b_{k+n} \) for
all \( k \).

p 394, l–9. When \( B \) is a vector with period \( n \), the Fourier transform of a convolution

p 395, ex. 2. of \( \omega \) is in the form \( jk \),

p 396, eq. (43). bold face equation number.

p 397, Step 3. \( \text{RFFT}(n/2, x_0, x_2, \ldots, x_{n-2}, 2m, y_0, y_1, \ldots, y_{n/2-1}) \).

p 397, Step 4. Call \( \text{RFFT}(n/2, x_1, x_3, \ldots, x_{n-1}, 2m, y_{n/2}, \ldots, y_{n-1}) \).

p 397, l11. Step 2 stops
p 398, Table: Level 1. $y_3 \leftarrow y'_3 - \omega^2 y'_3 = x_j - x_{j+4} - \omega^2 x_{j+2} + \omega^2 x_{j+6}$
p 402, l 7. However, $\omega^{n-1} = \omega^n \equiv 1 \pmod{m}$, so $\omega^{n-1}$ is the inverse we need.
p 406, l 13. $O(k)$ arithmetic operations are done, time $O(k^3)$
p 406, l 1. $O(k^2)$.
p 408, l 5. set of equations. Set $y_0$ to the determinant of the $a$'s (mod the current prime). The current prime is skipped if the determinant is zero.)
p 408, ex. 1. For your list of primes use 20,011 and 20,021.
p 409, l 10. we can try to find an algorithm that runs in a time equal to the lower bound.
p 413, ex. 3. at least $\left\lceil \log \left( \frac{n+k}{k} \right) \right\rceil$
p 415, l 11. the best algorithm
p 418, eq. (12). $C^+ = C \lor C^2 =$
p 418, l 5. $T(n) = O(n^3)$
p 419, l 14. $A_{12}(I \lor (A_{22})^+)A_{21}$.
p 419, eq. (17). $[A^+]_{11} = (A_{11} \lor A_{12}(I \lor (A_{22})^+)A_{21})^+$.
p 419, eqs. (18-20).

\begin{align*}
[A^+]_{12} &= (I \lor [A^+]_{11})A_{12}(I \lor (A_{22})^+) , \quad (18) \\
[A^+]_{21} &= (I \lor (A_{22})^+)A_{21}(I \lor [A^+]_{11}) , \quad (19) \\
[A^+]_{22} &= (A_{22})^+ \lor (I \lor (A_{22})^+)A_{21}(I \lor [A^+]_{11})A_{12}(I \lor (A_{22})^+) . \quad (20)
\end{align*}

p 420, Step 2. $T_2 \leftarrow I \lor T_1$.
p 420, Step 4. $[A^+]_{11} \leftarrow (A_{11} \lor T_3 A_{21})^+$
p 420, Step 5. $T_4 \leftarrow I \lor [A^+]_{11}$.
p 420, Step 9. $[A^+]_{22} \leftarrow T_1 \lor [A^+]_{21} T_3$.
p 422, l 10. to find the minimum
p 423, l 13. best known
p 430, ex. 5. Explain
p 431, l 12. choosing
p 432, ex. 1. Also consider $Q'_i(a, b) = Q_i(-a, b)$.
p 435, l 13. between $v_{p(1)}$ and $v_{p(n)}$.
p 435, l 21. can connect the component
p 436, fig. The third index for the $b$'s should be on the same level as the other indices.
p 436, l 6. came into a component through a node associated with $v_j$ and
p 437, l 5. returns to a different a node,
p 437, l 19. vertex cover for $G$,
p 438, l 4. is a tight bound are not,
p 441, l 23. Does there exist a Turing machine
p 445, eq. (3).

\[ F(x) = \left( \frac{1}{2} \right)^{10} \sum_{i \leq x} \binom{10}{i} . \quad (3) \]

p 445, l 9. the probability that it is less than 2 is $\left( \frac{1}{2} \right)^{10} + 10 \left( \frac{1}{2} \right)^{10}$; etc.
p 446, l 20. false hypothesis, i.e., to decide
p 448, l 10. A won 1640 out of 3200 games.
Thus, this way of analyzing the data suggests that method A might be better, but it does not provide strong support for the conclusion.

\[ F(x) = \text{Prob}(X \leq x), \]  
\[ H_n(x) = x - \frac{\cos(2\pi nx)}{2\pi n} \quad \text{for} \quad 0 \leq x \leq 1, \]  
\[ \phi_x(t) = 1 - \frac{1}{2}\sigma^2 t^2 + O(t^3), \]

because \( X - \mu \) has mean zero and variance \( \sigma^2 \), and the higher-order terms in the power series have size \( O(t^3) \). The random variable \( (X - \mu)/(\sigma\sqrt{n}) \) has the characteristic function \( \phi_x(t/(\sigma\sqrt{n})) \). Finally since \( Z_n \) is the sum of \( (X_i - \mu)/(\sigma\sqrt{n}) \), by applying eq. (26) \( n - 1 \) times, we get
\[ \phi_{Z_n}(t) = \left[ 1 - \frac{t^2}{2n} + O\left(\frac{t^3}{n^{3/2}}\right) \right]^n = \exp\left[ -\frac{t^2}{2} + O\left(\frac{t^3}{n^{3/2}}\right) \right]. \]

(This last result is from Exercise 4.2.2-7.) Note that in eq. (1) the big \( O \) term refers to the limit as \( n \) goes to infinity and \( t \) goes to zero. For any fixed \( t \) the sequence of characteristic functions approaches \( e^{-t^2/2} \), which is the characteristic function

\[ p_{t,e}(l) = (1 - b)\delta_{l1}, \]  
\[ p_{t,e}(e) = (1 - b)\delta_{e1}, \]  
\[ e_i = \frac{(26)^{i+1} - 1}{2b - 1}, \]  
\[ s_i = \sum_{e,t} e^2 p_{it}(e). \]
\[ s_i = \frac{1}{(2b - 1)^3} \left[ \frac{4b^4 - 2bp - 1}{4b - 2bp - 1} + \frac{2(3 - p)(1 - b)(2b - 1)^2}{(2b - 2p - 1)(p - 1)(2b + p - 2)} \right][2b(2 - p)]^{i+1} \]
\[ + \frac{2(2b - p - 1)}{p - 1} (2b)^{i+1} + \frac{p}{2b + p - 2} (4b)^{i+1} \] \( \text{or} \)
\[ \frac{2(1 - b)}{(2b - 1)^3} \left[ \frac{4b^4(1 - p)}{4b^2 - 2bp - 1} + \frac{(2b - 1)^3}{(4b - 2bp - 1)(1 - p)(2b + p - 2)} \right][2b(2 - p)]^{i+1} \]
\[ + (2b - 1) \left( \frac{1 + p}{1 - p} + 2i + 2 \right)(2b)^{i+1} + \frac{1 - p}{2b + p - 2} (4b)^{i+1} \]. \( \text{(113)} \)

p 477, eq. (128).

\( \frac{1}{x - x_j} = \frac{1/x}{1 - x_j/x} = x^{-1} \left( 1 + \frac{x_j}{x} + \frac{x_j^2}{x^2} + \cdots \right) \), \( \text{(23)} \)

p 495, ref. 9. *Fundamental Algorithms*

p 500. Interchange the numbers on references 80 and 81.

p 502, ref. 116. Physics

p 502, ref. 130. of List Update Rules,

p 503, ref. 135. van Leeuwen

p 503, ref. 136, 137. Jeffrey

p 510, eq. (3.209) \( x^{-i} \) means \( \frac{(-1)^i}{(1-x)^i} \).

p 513, eq. (1.84).

\[ \lim_{x \to b} \frac{f(x)}{g(x)} = \lim_{x \to b} \frac{f'(x)}{g'(x)} \text{ for } \lim_{x \to b} f(x) = \lim_{x \to b} g(x) = \infty. \] \( \text{(1.84)} \)

p 515, eq. (3.193)

\[ \sum_i \binom{n}{i} \binom{m}{i} = \binom{n + 1}{m + 1}. \] \( \text{(3.193)} \)

p 517, eq. (2.83)

\[ \sum_{1 \leq r < N} [\lfloor r \rfloor] = N[\lfloor N \rfloor] - 2^{\lfloor N \rfloor + 1} + 2. \] \( \text{(2.83)} \)

p 517, eq. (Ex. 4.5.4–4)

\[ \sum_{i \geq 0} i^{-2k} = \frac{(-1)^{k+1}B_{2k}(2\pi)^{2k}}{2!(2k)!}. \] \( \text{(Ex. 4.5.4–4)} \)

p 531, Eppinger, Jeffrey L., 483–485, 500.

p 532, Hamilton, William R.

p 533, Hermite, Charles
p 533, integral, Riemann
p 533, Internal path length, 280, 483–486.
p 536, Pure literal rule, 299, 433.
p 536, Pigeonhole principle
p 536, Poisson, Siméon-Denis